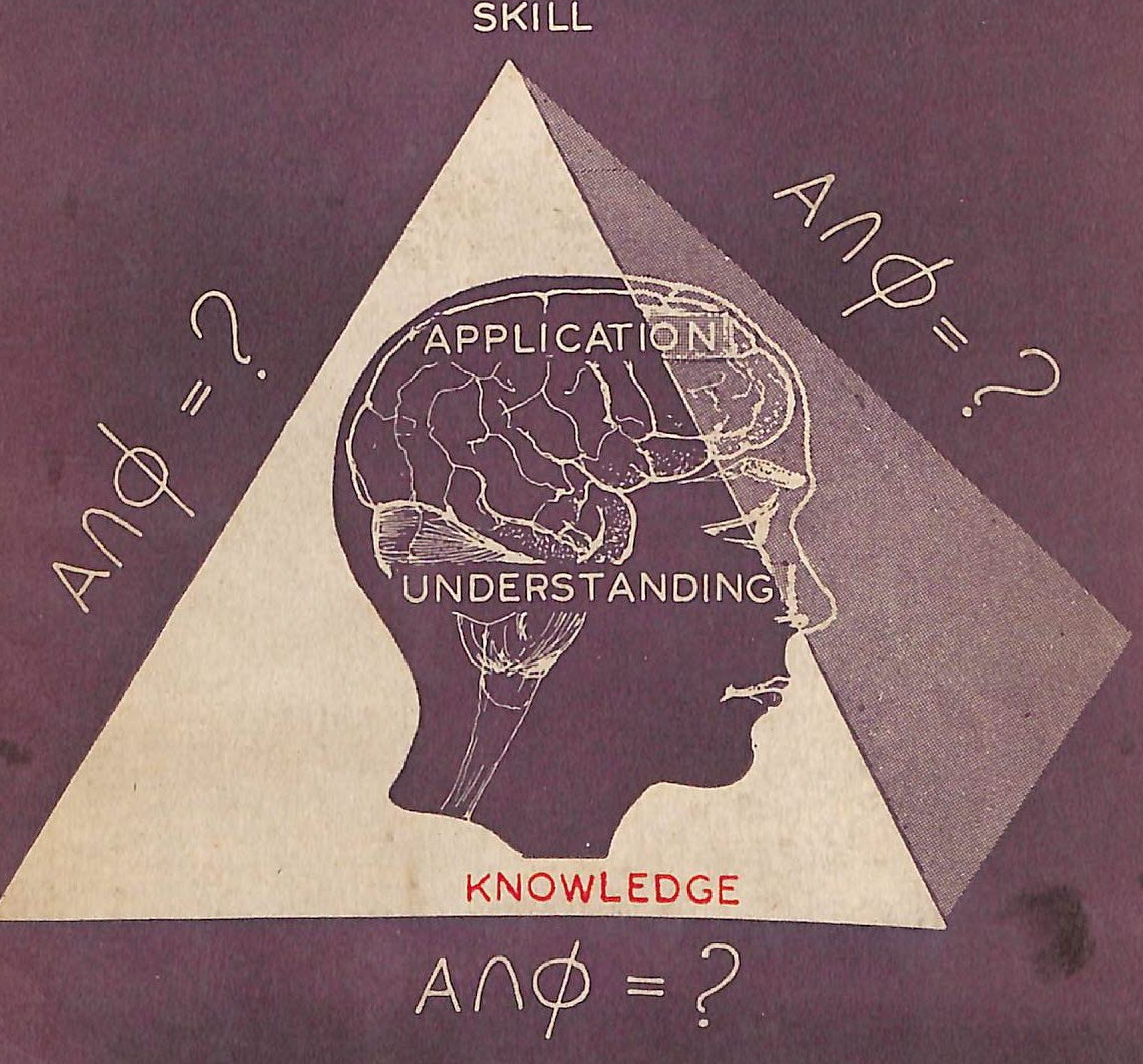
Objective-Based Test Items In MATHEMATICS



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Objective-Based Test Items in Mathematics for Class IX and Techniques of Framing Good Question Papers

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FOREWORD

This is the first book on objective-based test items in Mathematics for Class IX produced by the Department of Education in Science and Mathematics (DESM). This supplementary book in Mathematics which is intended mainly for teachers contains not only the test items of different objectives and different forms, but also a sample question paper, weightage to be given to different objectives, forms of questions and contents, preparation of blue prints, scoring key, marking scheme, question-wise analysis, etc., the knowledge of which is necessasy for becoming an efficient question-setter.

This book will make the teachers realise the fact that framing of question papers is not simply to collect questions of their choice from different books. Preparation of questions and question papers is an art. The present day question papers at many places suffer from many defects and, therefore, examinations based on these are defective in the sense that these can not evaluate the students' performance in the class as these are not the true measure of evaluation. It is, therefore, hoped that the present book will help the teachers to know some techniques of framing good question papers to enable them to evaluate the students' performance in the class properly.

The Department of Education in Science and Mathematics, NCERT will be grateful to the readers of this book for their reactions and any suggestion for its further improvement.

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PREFACE

This book is intended mainly for the teachers of Mathematics. The questions have been prepared on the basis of the syllabus and the text book prepared by NCERT. It has been observed that teachers get much difficulties in framing good question papers for testing different abilities of the students. This book has, therefore, been written to enable the teachers to frame good questions and question papers without much trouble.

Most of the questions in Chapters IV to XIII were prepared in a workshop held at Jadavpur University in Calcutta in March, 1979 by the expert and experienced teachers of schools, colleges and universities all over India.

A question bank in any subject is not the arbitrary collection of questions from different books. This is to be prepared systematically on some principles. We usually say that a particular question is difficult or easy without perhaps knowing which factor or factors are responsible for the difficulty level of a question. Prof. B.S. Bloom had classified different mental abilities of human beings starting from the lowest to the highest ability. Objective-based questions testing different abilities will naturally be of different types — those testing lower abilities will be easy and those testing higher abilities will be difficult. Although framing of questions on the basis of instructional objectives has many drawbacks, this is the most effective method of framing questions till today. I, therefore, chose this method for the preparation of this book. The method was explained to the participants in the workshop and they prepared the questions according to the guidelines. Out of many objectives, only four main objectives were selected for the preparation of this question bank. Depending mainly on the length of the answers of questions, I have further classified the questions as essay type, short answer type, etc. My plan was to prepare questions of four objectives mentioned in the book under each form of questions. It was later found that questions testing higher abilities like "Application" and "Skill" as prepared by the teachers were very few in number and hence in order to supplement it, most of the questions of "Application" and "Skill" types had to prepared by me. In order to explain the instructional objectives, different forms of questions and the procedure of framing a question paper, I have written three articles which are given in chapters I, II and III. A sample question paper has been prepared by me for the benefit of question-setters and this is given in Chapter XIV. At the end of the book, I have given the reference of some books, journals. etc. for enthusiastic teachers.

It may be pointed out here that the determination of the objective of a particular question is sometimes a matter of controversy, because the same question may appear to be of different objectives to different persons if they interpret it in different ways. A question, for example, may be a "Knowledge" type to a particular teacher who may think that the solution is known to the students, but the same question may be a question of "Understanding" type to another teacher who may think otherwise. Therefore, although much effort had been given in finalising the objectives of all questions, some readers may still have different opinions about the objectives of a question in the book. This, however, does not matter much, since our principal aim is to prepare questions of various types.

Finally, I like to give many thanks to the teachers for their significant contributions in framing questions as well as to Dr. B. Deokinandan and Dr.S.K. Singh Gautam, my colleagues, who assisted me in various ways e.g., in conducting the workshop in Calcutta, in finalising the materials prepared in the workshop, checking the answers of the problems and in final writing of the manuscript.

Any kind of suggestions from the readers for the improvement of the book will be gratefully acknowledged.

S.C. DAS
Editor

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CHAPTER I

INSTRUCTIONAL OBJECTIVES IN MATHEMATICS

1. Introduction

The word "Objective" is a broad term. Mathematics teachers can list many objectives of teaching Mathematics. For example, the objectives may be to enable the pupils to be familiar with mathematical terms, concepts and processes, use mathematical concepts and processes in their day to day life, appreciate the role of mathematics in every day life, understand, express and interpret symbolic representation with the manipulation thereof and so on. It may be noted that some of the objectives may not be amenable to easy testing through paper and pencil tests.

One of the important objectives of education is to know how far a student has learnt the content in a subject and how much intellectual skills have been developed in him.

Mathematics is a kind of language with its own vocabulary and grammar. This vocabulary consists of concepts, terms, facts, assumptions, symbols, etc. while the syntax relates to the principles, processes, functional relationships and the like. Knowledge and understanding of these and their application to new stituations have helped mankind to achieve tremendous progress in science and technology. These, therefore, should form the objectives of teaching mathematics. Besides, the subject-matter has been found suitable for developing certain intellectual abilities.

2. Classification of Instructional Objectives and Specifications

A number of models of classification of objectives has been developed. The most convincing of these in the recent years has been the taxonomical model of educational objectives developed by Prof. B.S. Bloom and his associates. It has adopted three primary aspects of mental life : cognitive, affective and psychomotor. The cognitive domain represents the intellectual area of human beings involving the development of the abilities of knowledge, understanding, application, skill, etc. It deals with the objectives relating to recall or recognition of knowledge and the development of intellectual abilities and skills. This domain is the most central to the work of much current test development. The affective domain includes objectives which describe changes in interests, attitudes, values and the development of appreciation. The psychomotor domain covers the region which includes muscular action and requires neuro-muscular co-ordination.

As regards mathematics teaching, we shall be concerned mainly with the objectives in the cognitive domain, because it is the intellectual area of human beings.

Specifications of instructional objectives of cognitive domain

The main objectives of the cognitive domain are (1) Knowledge (K) (2) Understanding (U) (3) Application (A) and (4) Skill (S). These objectives should be so stated that the teacher and the evaluator understand these easily. There may be aspects of an objective, that are not explicit enough in the statement of the objective. The work of identifying well-defined behavioural changes requires that objectives should be broken down into more specific ones to a limit where these stand fully clarified. It is, therefore, necessary to specify objectives in terms of expected changes in the students thinking, feeling and doing.

Some of the specifications of the objectives are mentioned below :

Objective : **Knowledge (K)**

The pupils acquire the 'Knowledge' of terms, concepts, symbols, definitions, principles, processes and formulae of Mathematics.

Specifications : The pupils

1.1 recall or reproduce terms, etc.

1.2 recognise terms, etc.

Objective : **Understanding (U)**

The pupils develop "Understanding" of terms, concepts, symbols, definitions, principles, processes and formulae of Mathematics.

Specifications : The pupils

2.1 give illustrations for terms, etc.

2.2 detect (and correct) errors in definitions, etc.

2.3 compare concepts, etc.

2.4 discriminate between closely related concepts.

2.5 classify data as per criteria.

2.6 find relationships among the given data.

2.7 translate verbal statements into symbolical statements and vice versa.

2.8 estimate the results.

2.9 interpret.

2.10 verify the results arrived at.

Objective : **Application (A)**

The pupils apply their knowledge and understanding of Mathematics to unfamiliar situations (or new problems). If the situation is familiar, it evokes mechanical reproduction, as in the case with the present day Mathematics examination. It is the new situation which involves real thinking and discovering on the part of the students.

Specifications : The pupils

3.1 analyse and find out what is given and what is required.

3.2 find out the adequacy, superfluity or relevancy of data.

3.3 establish relationship among the data.

3.4 select the appropriate method for solution of problems.

3.5 suggest alternative methods.

3.6 generalize (i.e. reason deductively).

Objective : **Skill (S)**

The pupils acquire skill in

(a) computation.

(b) drawing geometrical figures and graphs.

(c) reading tables, charts, graphs, etc.

(a) COMPUTATION

Specifications : The pupils

4.1 carry out oral calculations easily and quickly.
4.2 carry out written calculations easily and quickly.

(b) DRAWING OF GEOMETRICAL FIGURES AND GRAPHS

Specifications : The pupils

4.3 handle geometrical instruments easily and quickly.
4.4 measure length etc. accurately.
4.5 draw freehand figures easily.
4.6 draw figures to the scale.
4.7 draw figures accurately.

(c) READING TABLES, CHARTS, GRAPHS, ETC.

Specifications : The pupils

4.8 read tables accurately and quickly.
4.9 interpret graphs.

It may be noted that Knowledge is the lowest ability, Understanding is of higher ability than Knowledge and Application is higher than Understanding. The Skill ability mainly involves in computation, reading graphs, etc. as mentioned above.

In the present question bank in Mathematics, a large varieties of questions of each objective have been prepared. It can be observed that questions of higher abilities like Application and Skill will be more difficult than questions of other abilities.

3. Some examples of Objective-based questions

Some examples of objective-based questions with specifications of the objectives are given below.

Q 1. Fill up the blanks with the appropriate answer given in the bracket :
The point of intersection of three medians of a triangle is called the —— (in-centre, centroid, circumcentre)

The above question tests the memory of a student to determine whether the student has memorised the definition of "centroid" of a triangle or not. Hence, it is a "Knowledge" type question. Specification is 1.1.

Q 2. Give a tick mark to the correct answer given in (A), (B), (C) or (D) below for the following :
If the radii of two circles are in the ratio 2 : 3, then their areas are in the ratio
(A) $\sqrt{2} : \sqrt{3}$
(B) 2 : 3
(C) 4 : 9
(D) none of the above

The above problem is of "Understanding" type. Specification is 2.4.

Q 3. Give a tick mark to the correct answer given in (A), (B), (C) or (D) below for the following :
In a triangle ABC, AB and AC are produced to E and F respectively and OB and OC are the bisectors of angles EBC and FCB. If $\angle A = 40°$, then $\angle BOC$ is
(A) 20°
(B) 40°
(C) 50°
(D) 70°

The above problem is of "Application" type. Specification is 3.3.

Q 4. Construct a triangle ABC in which $\angle B = 60°$, BC = 6.9 cm. and the perpendicular AD on BC is 4.2 cm.

The above problem is of "Skill" type. Specifications are 4.6 and 4.7.

4. Some comments on instructional objectives

From the practical point of view, it should be pointed out here that a teacher lacking adequate knowledge of theories of learning mathematics and relevant areas in psychology may find difficulty in classifying questions according to instructional objectives. It is possible that teachers of mathematics make distinctions which psychologists would not make in classifying or studying human abilities. To a mathematician, confusion may arise to determine whether a particular question will test the ability "Understanding" or "Application" of a student, but we should not spend too much time to settle this controversy because our main aim is to frame questions of different types. Whenever confusion arises to determine the objectives of a question, one should apply his best judgement to decide it. The determination of the objective of a question based on Bloom's taxonomy varies from person to person. A question, for example, may appear to be of "Knowledge" type to a teacher who may think that the answer of the quesrion is known to the students, but the same question may appear to be of "Understanding" type to another teacher if he thinks that the question, is completely unknown to the same students. Mathematicians should, therefore, follow the specifications of each objective very carefully and determine the objective of a question as correctly as possible according to their best judgement.

CHAPTER II

DIFFERENT FORMS OF QUESTIONS

The form of a question is determined by the length of the answer required, time needed to answer it and the credit points of the question. Depending upon the objective and content, we may select different forms of questions to test a particular learning outcome. Most commonly used tools of evaluation are the written tests which are generally of two types : (1) Free response questions and (2) Fixed response questions. In a free response question, an examinee formulates and organises his own answer. This type of tests is very much in use in the prevailing system of examination. In a fixed response question, the answer is definite. This type of tests is being encouraged day by day due to its objectivity in scoring. However, both types of tests have considerable value when properly used. Free response questions are of two types : (1) Essay type and (2) Short answer type.

Essay type questions

A question which requires a comparatively lengthy answer to a problem say, more than one or two sentences, is called an essay type qnestion. There is no single correct response. Accuracy and quality of response can be evaluated by a person who knows the topic. These questions require the candidate to select relevant facts, organise them and write the answer in his own words.

Essay type questions have been repeatedly criticised by those who are interested in the scientific measurement of achievement, but little effort has been made to improve their reliability in spite of the fact that are widely used by classroom teachers. If properly constructed, they can measure important outcomes of learning which can not be measured otherwise. Higher mental processes like ability to organise, integrate, summarise, etc. are better evaluated by essay-type questions.

For constructing better essay type questions, the following points should be kept in mind :

(a) Essay type questions should be set to test only those instructional objectives which cannot be tested so easily by other forms.

(b) Each question should test the specific mental processes or learning outcome as indicated by the objectives in view.

(c) Questions should be framed in such a way that their meanings are clear to the examiners.

(d) Questions should be constructed in such a manner that the scope of the expected answer is clear to all.

(e) Directional words like "what do you know of", "give an account of", "write short notes on", etc. should be avoided as far as possible or should be clearly defined to avoid vagueness of answers and consequent variations of markings by different examiners.

(f) Maturation and level of the examinees should be considered while framing an essay type question. Length and nature of an answer may differ from class to class.

(g) Marks should be clearly allocated part-wise whenever there are more than one part in the same essay type question.

Short answer type questions

A question in response to which a student is expected to give an answer in about 3 or 4 lines is generally called a short answer type question. Such a question should ordinarily carry 2 to 4 credit points and should take 3 to 6 minutes to be answered by a student. In fact, limits of short-answer type questions cannot be clearly demarcated, still we have mentioned some criteria for the sake of convenience. It may be noted that a question whose answer may be a simple diagram is also considered to be a short answer type question.

It has been observed that essay type questions suffer from lack of objectivity and reliability while objective type questions, as explained later, are often inadequate to test certain aspects of growth. Short answer type questions are a good via-media between the two extremes and serve a useful purpose in overcoming the shortcoming of either type, if understood and framed properly by the teachers. Such type of questions can be used profitably in our external and internal examinations. These questions also ensure better reliability, because such questions are more definite and specific and can be scored more objectively. These also cover more syllabus as a number of such questions can be made in place of one essay type question. Therefore, the short answer type questions draw upon a far wider range of a pupil's achievement than do the usual essay questions. Such questions ensure adequate sampling and thereby improve the validity. Teachers can easily acquire the necessary competence to frame such questions. It is, therefore, desirable that such questions in the examinations should be encouraged to improve its validity and reliability.

Fixed response type questions are of two types : (1) Very short answer type and (2) Objective type.

Very short answer type questions

A question in response to which a student is expected to give an answer in a word or a phrase is called a very short answer type question. In mathematics, by a word or a phrase is generally meant a group of symbols or figures (numbers), etc. A very short answer type question generally carries 1 mark and each student is expected to take 1 to 3 minutes to answer it.

Objective type questions

An Objective type question is one where the alternative answers are given and the students have to indicate the correct answer by some signs such as a tick mark. Like a very short answer type question, an Objective type question generally carries 1 mark and each student is expected to answer it in 1 to 3 minutes.

Objective-based and Objective type questions are often confused with each other. When a question is framed with deliberate attention to the objective which we want to measure, it is an objective-based question. But when a question is designed to provide a measure of a student's achievement which is objective rather than subjective, we call it an Objective type question. Therefore, every question will be objective based but an objective type question is of a definite form as described above.

Objective type questions can further be classified as follows :

(1) True false type (2) Matching type (3) Sentence Completion type and

(4) Multiple Choice type.

Multiple Choice type questions can again be classified as follows :

(1) Only correct answer type and (2) Only wrong answer type In literature or other subjects, other types of Multiple Choice questions known as (3) Best answer type and (4) Mixed answer type or Master-list type can also be framed. In Mathematics, such types of questions [i.e. types (3) and (4)] are very rare.

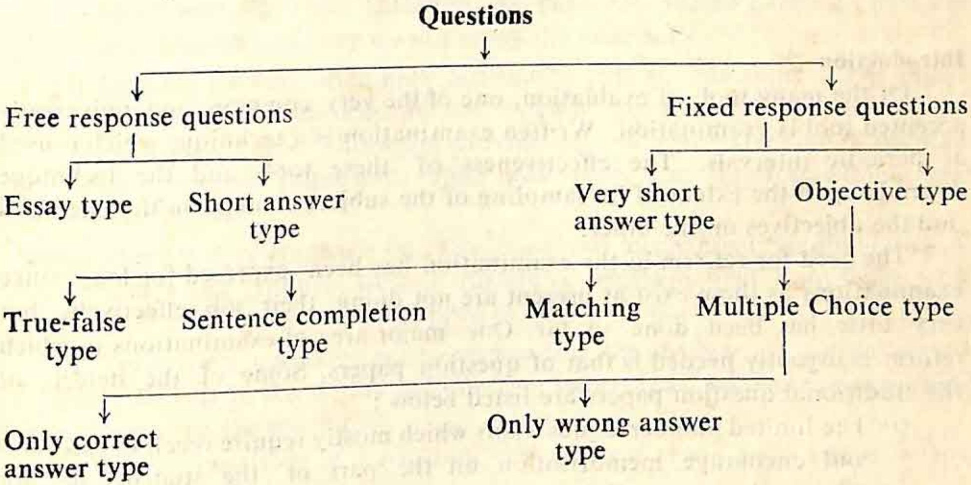
In True false type questions, some statements are given to be correct and some are incorrect. Students are generally asked to put "T" against the correct statements and "F" against the incorrect statements. In Matching type questions, generally two columns of statements or elements are given. Students are asked to match the statements or elements in column I with those in column II. In Sentence Completion type questions, one blank space in a statement is provided. Students are asked to fill up the blanks with appropriate answers which are given within a bracket. Generally, 2 to 4 answers are supplied and one of these is the correct answer. A Multiple Choice type question is composed of a stem in the form of a statement or a question followed by four alternative choices.

Following points should be kept in mind while structuring a good Multiple Choice question :

1. Be sure that only one choice out of four is correct and the choices do not overlap with each other.
2. The alternatives should not contain anything which could have been put in the stem.
3. Give 4 or at most 5 choices, but not more than 5.
4. Both correct and incorrect choices should be homogeneous in their mode of expression, length and other external characteristics. The distractors should represent errors commonly made of the students who are to be tested. (Note that any of the incorrect choices in a Multiple Choice question is called a distractor).
5. Avoid making the correct response consistently longer or consistently shorter than incorrect choices.
6. Arrange the choices in logical order, if one exists. Otherwise, these should be arranged in alphabetical order,

7. See that the position of the correct answer in the series is chosen entirely at random.
8. Make all choices grammatically consistent with the form of the question. See that the uses of singular and plural numbers, tenses, etc. do not serve as clues to the students.

The following table gives the classification of different forms of questions:



CHAPTER III

DESIGN OF A GOOD QUESTION PAPER

Introduction

Of the many tools of evaluation, one of the very common and universally accepted tool is examination. Written examination is a technique which is used at periodic intervals. The effectiveness of these tools and the technique depends upon the extent of its sampling of the subject content on the one hand and the objectives on the other.

The need for reform in the examination has been expresed for long, since examinations as these exist at present are not doing their job effectively, but very little has been done so far. One major area of examinations in which reform is urgently needed is that of question papers. Some of the defects of the traditional question papers are listed below :

(i) The limited number of questions which mostly require recall of informa- and encourage memorisation on the part of the students do not adequately cover all the topics of the syllabus or different aspects of a topic and do not test the higher abilitics like Understanding, Application and Skill.

(ii) Directional words like "Explain", "Interpret", "What do you know of", "Give an account of", "Write short notes on" etc. very often make a question paper vague. The students very often can not determine what answers are to be given. The examiners themselves are not clear about the quality and nature of the answers required by the students. Marks allotted by different examiners for similar answers, therefore, vary appreciably and hence evaluation of student's performance in the examination is defective.

(iii) Most of the problems in a question paper are generally stereo-typed textbook type of problems and only essay type questions are asked. As a result these questions are inadequate to assess the real abilities of students in problem solving. Since essay type questions may be very few in number, large portions of the syllabusare left uncovered. This leads to the guessing of expected questions and preparation of set answers for them becomes the main learning effort on the examinees.

(iv) Questions are very often repeated from the question papers of earlier years and students, therefore, prepare the solved papers to pass the examination and they do not study the textbooks.

(v) Options are very often provided stating that all questions carry equal marks and that the students may attempt any five or six out of ten or eleven questions. This again provides students with freedom to prepare only selected important topics and leave out the rest without any serious risk.

(vi) There is over-emphasis on calculation and mechanical manipulation of figures and symbols. Certain aspects of mathematical knowledge and their application are not tapped at all by the questions.

(vii) Some times questions are so framed that the answers are not unique.

These defects in the traditional question papers can be overcome through the following specific measures :

(a) Prepare questions of a question paper such that these can test different abilities of a student.

(b) The questions should be framed in simple and unambiguous language so that students may be clear about what they have to write as an answer to a particular question.

(c) Ask many short answer type and very short answer type questions in place of essay type questions only. It will provide the paper setters an opportunity to ask questions from the whole course and discourage pupils' habit of making selective study. A few essay type questions may also be asked.

(d) Avoid "Do any six of the following" type of options and, if necessary, give options within the question itself. This will discourage selective study of the pupils.

(e) Avoid using the directional words as mentjoned in (ii) above.

(f) Solve each question in all possible methods and be sure that you get the same answer or answers in whatever method you solve it.

Setting of a good question paper

The construction of a good question paper requires a great deal of planning prior to the actual writing of questions. Good questions do not just happen nor are these the results of a few moments of high inspiration or exaltation. On the contrary, the process is calm, deliberate and time consuming. The defects in the written tests of today might help in planning a good question paper, but the real guidance will have to be taken from the criteria of a good test which compromise the validity, reliability, objectivity and practicability. Based on these, we can see that the following are the major steps in the planning of a question paper :

Preparation of the design of a question paper

For public examination, the design of a question paper is generally prepared by the Board, but in case of teacher-made test, the teacher will have to prepare it for himself. The design of a question paper gives the details of some basic requirements for setting a good question paper. It also mentions certain policy decision regarding the various items discussed below :

Weightage to objectives

The paper-setter must see that all the objectives testable through the written examination get their due share and their respective weightages are based on national and social needs, nature of the subject ond the level of the student. They should also test as many specifications under each objective as possible. Decisions regarding allotment of an adequate weightage to the outcome or abilities which we expect to test will depend on a number of factors : (1) the nature of the content i.e., whether it is amenable to testing Knowledge more than Understanding or vice versa or any other outcomes are capable of being tested ; (2) the method in which these units or topics have been handled in the class i.e., whether the teacher has aimed at developing or stressing any one particular ability or he has given equal importance to develop all the abilities ; (3) the time taken in developing these abilities through the class and other learning activities etc. and (4) the ability level of students on whom the test is to be administered.

In competitive examinations, more weightage is given on questions of higher abilities whereas in Board's examination, less weightage is given on questions of higher abilities.

Weightage to the content

The paper setter has to make a decision about the weightage to be given to different topics or sub-topics of the subject depending upon their importance and the time spent on them in the class. No topic should, however, be completely ignored.

Weightage to forms of questions

In order to have more valid and reliable question papers, we must introduce more short answer, very short answer and objective type questions. Only a few essay type questions can be included depending on the duration of the examination. The paper-setter must decide the weightage to be given to different forms of questions. This will depend on the suitability of each form of questions of a particular objective and the content area.

Blue print of a question paper

Planning of a question paper in steps or stages as indicated above can be given a more concrete shape by the development of a three-dimensional chart called a Blue Print. When the weightages to different objectives, content areas and the forms of questions have been decided and also the number of questions to be asked is settled, the paper setter has to decide how all tho questions should be distributed over different objectives and content areas so as to have the weightages decided above. The three dimensions of the blue print consists of content areas in horizontal rows and objectives and forms of questions in the vertical columns. Once the blue print is ready, the paper-setter can select a frame item to fit into the blue print.

Preparation of scoring key and marking scheme of a question paper

Scoring key should be prepared by the question setter in order to check the setting of the questions, answers to which are not definite or clear even to the question-setter. A detailed scheme of marking will go a long way in insuring greater objectivity and reliability of scoring.

Preparation of question-wise analysis

Preparation of question-wise analysis plays an important part in the setting of a question paper. This provides an excellent check for all the different aspects of the question paper.

In question-wise analysis, we show the allotment of marks to various objectives, areas of content, forms of questions, time allotted to different questions, etc. When these are checked against the blue print, then this analysis determines whether the question paper has been set to specification or not.

Unit test

A unit test is a test based on one unit or topic of the syllabus whereas a question paper is a test based on the whole syllabus. Preparation of a unit test, therefore, involves the same steps as that of a question paper as discussed above.

Sections

When all the four types of a questions viz. essay, short answer, very short answer and objective types of questions are included in a question paper, it is sometimes desirable to put them in two sections — one consisting of objective type and very short answer type of questions and the other of the essay and short answer type of questions.

Conclusion

Based on the above discussion, a model question paper has been prepared and is given in Chapter XIV. This will give some guidelines to the question-setters to prepare any unit test or a question paper.

CHAPTER IV

UNIT I : REVIEW OF THE NUMBER SYSTEMS

FORM : **Essay Type**

Objective : KNOWLEDGE

1. Distinguish between prime numbers and composite numbers by giving definitions and examples of each kind. Can an even number be ever prime? If so, write the number.
2. Is $\sqrt{2}$ an irrational number? Justify your answer.

Objective : UNDERSTANDING

3. For positive integers x,y,z, if x is a factor of both y and z, show that x is also a factor of $y+z$.
4. Show with examples and proper reasonings, the commutativity for the operations of addition, subtraction, multiplication and division over N.
5. Is $3+\sqrt{5}$ an irrational number? Justify your answer.
6. Let x and y be natural numbers and let $x * y=(x-1)(y-1)-5$.
   (i) Evaluate $5 * 6$.
   (ii) Is the * commutative?
   (iii) Is the * associative?
   (iv) Is N closed under *?
   Justify your answers.

Objective : APPLICATION

7. If $\frac{a}{b} = \frac{c}{d}$; find the values of
$\frac{ma - nc}{mb - nd}$ and $\frac{pa + qc}{pb + qd}$ where m,n,p,q
are any numbers such that $mb - nd \neq 0$
and $pb + qd \neq 0$. Hence prove that
$\frac{ma - nc}{mb - nd} - \frac{pa + qc}{pb + qd} = 0$

8. Prove that if the square of an odd natural number is divided by 8, then the remainder will always be 1.

Objective : SKILL

9. Consider the number :
$$5378091^{985798} + 3987081^{73580} - 1001236^{100000} + 703985^{98735} - 16^4 + 359$$
   (i) Write the digit in the unit place of the above number.
   (ii) Is the number divisible by 5?
   (iii) Is the number divisible by 2?
   (iv) Represent 1/3rd of the digit in the unit place of the given number on the number line.

10. If $a = \frac{\sqrt{5} + 1}{\sqrt{5} - 1}$ and $b = \frac{\sqrt{5} - 1}{\sqrt{5} + 1}$, find
the value of $\frac{a^2 + ab + b^2}{a^2 - ab + b^2}$.

FORM : **Short Answer Type**

Objective : KNOWLEDGE

1. State when a pair of positive integers is said to be co-prime. Give one example.

2. Answer the following questions and in each case state the rule which you have used to find the answer.
   a) Is 10101 divisible by 3?
   b) Is 7668 divisible by 6?
   c) Is 3105 divisible by 9?

Objective : UNDERSTANDING

3. Find the values of $|-x| - |y|$ when
   (i) $x = -2$ and $y = -3$
   (ii) $x = 2$ and $y = 3$.

4. State the steps to show that H.C.F. of two prime numbers is 1.

5. Let a and b be two positive real numbers and let a*b denote the positive real number $\sqrt{ab}$. Find the values $4*(8*2)$ and $(4*8)*2$ and state whether the operation * is associative or not.

6. State with reasons the error(s) in the following statements marked with step 1, step 2, etc., which prove an absurd result :
   We know that
   $x^2 - x^2 = x^2 - x^2$ (Step 1)
   or, $x(x-x) = (x-x)(x+x)$ (Step 2)
   or, $x = x+x$ (Step 3)
   or, $x = 2x$ (Step 4)
   We now divide both sides by x. (Step 5)
   $\therefore\ 1 = 2$ which is absurd.

7. We know that if $x^a = y^a$, then $x = y$.
   Now, since $1^0 = 2^0 = 3^0 = 1$,
   hence $1 = 2 = 3$.
   State with reasons the error in the statement(s) which proves an absurd result.

8. Find the H.C.F. and L.C.M. of the pair of numbers 24 and 36 and hence find the relationship between the product of two numbers, their L.C.M. and H.C.F.

9. Can the greatest common factor of some whole numbers be ever (i) equal to (ii) greater than or (iii) less than the least common multiple of these whole numbers? Give an example in each case.

10. Give reasons to prove that the product of three consecutive natural numbers will always be divisible by 6.

Objective : APPLICATION

11. Let a and b be real numbers. Write down the expressions for $a \odot b$ and $b \odot a$ where $\odot$ means the operation of multiplying the second number by the first number increased by 2. By simplifying $(a \odot b) - (b \odot a)$, state what replacement for a and b would make $a \odot b = b \odot a$ true.

12. If a and b are co-prime, find two other integers u and v so that
    $$au + bv = 1$$
    where $a = 3$, $b = 5$ and the sum of u and v is 1. Is there any superfluous statement or data in the above problem? If so, state it.

13. State the values of x for which each of the following statements is true :
    (i) $(x+4)^2 = x^2 + 8x + 16$
    (ii) $x + 1 = 1$
    (iii) $x + 3 = 2$

Objective : SKILL

14. Arrange the following in descending order :
    $\frac{1}{5}(0.065)^{0{\cdot}5}$, $-(1.1)^2$, $(0.25)^{-1/3}$

FORM : **Very Short Answer Type**

Objective : KNOWLEDGE

1. Find out the rational and irrational numbers from the following :
   $5, \frac{3}{2}, \sqrt{7}, \sqrt{12}, 0.023, -1, 2 + \sqrt{3}$

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Date..........................
Acc. No......................

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Calcutta

Objective : UNDERSTANDING

2. For what values of a number x ($\neq 0$), its reciprocal will be greatest than it?

3. For what values of a number x ($\neq 0$), its reciprocal will be less than it?

4. For what value(s) of x ($\neq 0$), $\frac{1}{x} = x$?

5. Is the sum of two prime numbers always even? If not, give an example where the sum may be odd.

6. If one fraction has a larger numerator than the second fraction, then is the number represented by the first fraction always larger?

7. If we divide a positive integer by another positive integer, will the resulting number be an integer or a rational number?

8. Can there be a finite number of rational numbers between 1/2 and 1/4?

9. If $ab = 0$, what possible conclusions can you draw about a and b when a and b are real numbers?

Objective : APPLICATION

10. What is the value of $a^3 + b^3$ if the sum of $\frac{a}{b}$ and its reciprocal is 1?
($a \neq 0$, $b \neq 0$)

Objective : SKILL

11. What is the digit in the unit place of the number 9^{9^9} ? Is this number divisible by 5?

FORM : **Objective Type**

(i) *Multiple choice type*

In each of the questions (No. 1 to 6), there are four possible answers marked (A), (B), (C), and (D). Only one of these answers is correct. The

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letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right-hand margin.

Objective : **Knowledge**

1. If $(a, b) = \frac{a}{b}$, then $(\frac{a}{b}, \frac{b}{a})$ is equal to

(A) $\frac{a^2}{b^2}$

(B) 1

(C) $\frac{b^2}{a^2}$

(D) none of the above ( )

Objective : UNDERSTANDING

2. A number is irrational if

(A) it is of the form $\frac{p}{q}$, where p and q are non-zero integers

(B) it can be expressed as non-terminating decimals

(C) it can be expressed as non-recurring decimals

(D) it can be expressed as non-terminating non-recurring decimals ( )

Objective : APPLICATION

3. If the digit 1 is placed after a two-digit number whose ten's digit is t and unit's digit is u, the new number is

(A) $10t + u + 1$

(B) $100t + 10u + 1$

(C) $t + u + 1$

(D) none of the above ( )

4. The sum of any two irrational numbers is

(A) sometimes a rational number and sometimes an irrational number

(B) always an irratoinal number

(C) always a rational number
(D) always an integer ( )

5. The product of any two irrational numbers is
   (A) always an irrational number
   (B) always a rational number
   (C) always an integer
   (D) sometimes a rational number and sometimes an irrational number ( )

Objective : SKILL

6. If $x = 2 + 3^{1/3} + 3^{2/3}$, then $x^3 - 6x^2 + 3x$ is
   (A) 2
   (B) 0
   (C) $3^{1/3} + 3^{2/3}$
   (D) none of the above ( )

(ii) *True false type*

In the following statements (No. 1 to 21), write 'T' against each true and 'F' against each false statements at the end of each statement.

Objective : KNOWLEDGE

1. Every rational number can be represented by a point on the number line.
2. $3\frac{1}{7}$ is a rational number.
3. 2 is not a prime number.
4. All odd numbers are prime.

Objective : UNDERSTANDING

5. The square of an irrational number is always an irrational number.
6. The square of an irrational number is always a rational number.

7. The square of an irrational number is sometimes an irrational number.
8. All rational numbers between 15 and 18 are finite.
9. The sum of two rational numbers having the same numerators may be formed by adding their denominators.
10. 7/3 is greater than 2/5.
11. -13 is greater than -19.
12. If the radius of a circle is a rational number, its area is given by a number which is rational.
13. If the radius of a circle is a rational number, its circumference is given by a number which is irrational.

Objective : APPLICATION

14. $-3x > -7x$ when $x > 0$
15. If a and b are two rational numbers such that $a^2 = b^2$, then a is always equal to b.
16. If a and b are two positive rational numbers such that $a^3 = b^3$, then a is always equal to b.
17. If $ax = bx$ where a and b are rational numbers. then a is always equal to b.

Objective : SKILL

18. $\sqrt[3]{10} > \sqrt[4]{15} > \sqrt{3}$
19. $\sqrt[4]{15} > \sqrt[3]{10} > \sqrt{3}$
20. $\sqrt{3} > \sqrt[4]{15} > \sqrt[3]{10}$
21. $\sqrt{3} < \sqrt[4]{15} > \sqrt[3]{10}$

(iii) *Matching type* :

Objective : KNOWLEDGE

1. Certain numbers are given in column I and the names by which they are called are given in column II. Match those in column I with those in column II in the manner such as (N)−(X), where N denotes an integer and X denotes an English alphabet.

Column I	*Column II*
(1) $3\sqrt[3]{9}$	(A) an additive inverse of 9
(2) 9^{-1}	(B) a positive integer
(3) -9	(C) a reciprocal of 9
(4) 9	(D) an irrational number

2. The nature of some numbers is stated in column I and the names by which these numbers are called are given in colum n II below. Match those in column I with those in column II in the manner such as (1)−(C), (2)−(A), etc.

Column I	*Column II*
(1) A pair of numbers whose H.C.F. is always 1.	(A) composite
(2) Numbers which are never divisible by 2.	(B) prime
(3) Numbers which are always divisible by 2.	(C) co-prime
(4) A number which has factors other than 1 and the number itself.	(D) even
(5) A number which is divisible by 1 and itself and never divisible by any other numbers.	(E)

CHAPTER V

UNIT II : SETS AND MAPPINGS

FORM : Essay Type

Objective : KNOWLEDGE

1. Prove that $A' \cap B' = (A \cup B)'$

2. Let $A = \{x, y, z\}$, $B = \{x, y\}$, $C = \{x, y, t\}$, $D = \{z, t\}$ and $E = \{t\}$.

   State which of the following statements are true or false and give reasons in each case :

   (i) $B \subset A$ (ii) $D \neq C$ (iii) $C \supset E$ (iv) $D \not\supset E$ (v) $D \subset B$
   (vi) $D = A$ (vii) $B \not\subset C$ (viii) $E \subset A$ (ix) $E \not\subset B$ (x) $x \subset A$
   (xi) $x \in A$ (xii) $\{x\} \in A$ (xiii) $\{x\} \subset A$.

Objective : UNDERSTANDING

3. In a cricket team of 16 players, 10 are selected as batsmen and 8 as batsmen and bowlers both. What is the number of players who are selected as bowlers only ?

   Verify the result by Venn diagram also.

Objective : APPLICATION

4. Using the formula
   $n(A \cup B) = n(A) + n(B) - n(A \cap B)$,
   prove that

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(C \cap A)$
$+ n(A \cap B \cap C)$.

Objective : SKILL

5. Graph the function $f : x+2 \rightarrow |x|, x \in R$.

FORM : Short Answer Type

Objective : KNOWLEDGE

1. Write down the elements of each of the following sets and find the pair(s) of equal sets, if any:
$A = \{0\}$, $B = \{x \mid x > 15 \text{ and } x < 5\}$,
$C = \{x \mid x - 5 = 0\}$ and $D = \{x \mid x^2 = 25\}$.

2. Write all the subsets of the set $\{a, b, c\}$

3. Find the unions and intersections of
   (i) $A = \{1, 3, 7, 9\}$ and $B = \{0, 2, 4, 6, 8\}$
   (ii) $A \{0, 1, 2\}$ and $B = \{1, 2, 3\}$
   (iii) $A = \{x \mid x \text{ is a multiple of 5 and } 0 < x \leqslant 25)$,
   $B = \{x \mid x \text{ is a multiple of 3 and } 1 \leqslant x \leqslant 15\}$

4. Given $A = \{-2, 3, 8\}$
and $B = \{0, -5\}$
Determine A x B and draw its graph.

5. If $n(A) = 8$, $n(B) = 5$ and $n(A \cap B) = 4$, find $n(A \cup B)$.

Objective : UNDERSTANDING

6. If $A = \{x \mid x \text{ is a positive even integer and } x < 100\}$,
$B = \{x \mid x \text{ is a multiple of 5 and } x < 110\}$
and $C = \{x \mid x \in N \text{ and } 25 < x \leqslant 100\}$,
find (i) $(A \cap B) \cap C$
and (ii) $A \cap (B \cap C)$.

7. If $X = \{ a, b, c \}$
   $Y = \{ 1, 2 \}$
   $Z = \{ 1 \}$,
   write down the following cartesian products :
   (i) $X \times Y$ (ii) $Y \times X$ (iii) $X \times Z$.
   Is $X \times Y = Y \times X$?

Objective : APPLICATION

8. Given that $f(x) = \frac{1}{x} + 3x$. Find $f(\frac{1}{x})$, where $x \neq 0$.

9. If $f(x + 2) = x$, find $f(x - 2)$ and $f(\frac{1}{x})$ where $x \neq 0$.

10. If $f(x+2) = 5$, show that $f(-x) + f(2x) = 10$.

Objective : SKILL

11. Given that $f(x + 1) = -\frac{1}{x}$ and $g(x + 2) = \frac{x + 1}{x + 2}$. Calculate $xf(x)\,g(x)$. Hence, can you find the value of $f(1)\,g(1)$? Give reasons.

FORM : **Very Short Answer Type**

Objective : KNOWLEDGE

1. If a set A contains 11 elements and a set B contains 5 elements such that $B \subset A$, how many elements are there in each of the following sets?
   (i) $A \cup B$ (ii) $A \cap B$.

2. For any set A, are the sets $A \cap A$ and $A \cup A$ the same ?

3. State the following sets in words :
   (i) $A \cap B'$ (ii) $(A \cap B) \cap C$.

4. State which of the following collections are sets and which are not :
   - (i) A collection of short boys in your class.
   - (ii) A collection of books written by Rabindra Nath Tagore.
   - (iii) A collection of all books on Set Theory in your school library.
   - (iv) A collection of all long rivers in India.
   - (v) A collection of good books on Mathematics in your school library.
   - (vi) A collection of all even natural numbers.

5. Rewrite the following sets in the Roster form (tabular form) :
   - (i) A is the set of all odd integers between 50 to 80.
   - (ii) S is the set of all the vowels in the word 'arrangement'.

6. State which of the following sets are finite and which are infinite :
   - (i) $\{ x \mid x \in N \text{ and } (x - 1)(x - 2) = 0 \}$
   - (ii) $\{ x \mid x \in N \text{ and } x^2 = 4 \}$
   - (iii) $\{ x \mid x \in N \text{ and } 2x - 1 = 0 \}$
   - (iv) $\{ x \mid x \in N \text{ and and } x \text{ is prime} \}$
   - (v) $\{ x \mid x \in N \text{ and } x \text{ is odd} \}$

Objective : UNDERSTANDING

7. If $A = \{ 2, 4, 6, 8 \}$ and $C = \{ 2, 3, 4, 5, 6, 8 \}$, write all non-empty sets B such that $B \subset C$ and $B \cap A = \phi$.

8. If $A = \{ a \}$ and $B = \{ b, c \}$, write down a relation between a, b, c, such that $A = B$.

Objective : APPLICATION

9. If $f(|x|) = |x| + 1$, what is the value of $f(x) + f(-x)$?

Objective : SKILL

10. A function $f(x)$ is said to be odd if $f(-x) = -f(x)$. If $f(x)$ is an odd function such that $f(x) - f(-x) = 2x$, then what is the value of $\frac{f(-1)}{f(1)}$?

FORM : Objective Type

(i) *Multiple choice type* :

In each of the questions (No. 1 to 15), there are four possible answers marked (A), (B), (C) and (D). Only one of these answersis correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right-hand margin.

Objective : KNOWLEDGE

1. If $A = \{ y \mid y = 1 - (-1)^x, x \in I \}$, then A is a
   (A) finite set other than a null set
   (B) null set
   (C) infinite set other than the set of all even integers
   (D) set of all even integers ( )

2. If $n(A) = p$, $n(B) = q$, then $n(A \times B)$ is equal to
   (A) pq
   (B) $p + q$
   (C) $p + p + 1$
   (D) none of the above ( )

3. If $A = \{ x \mid 0 < x < 40, x \text{ is a multiple of } 3 \}$
   $B = \{ x \mid 0 < x < 40, x \text{ is a multiple of } 4 \}$
   $C = \{ x \mid 0 < x < 40, x \text{ is a multiple of } 5 \}$,
   then $n(A) + n(B) + n(C)$ is equal to

(A) 40 (C) 37
(B) 30 (D) 29

4. If n (A) = p, then the number of all the subsets of A will be
   - (A) p
   - (B) p^2
   - (C) 2^p
   - (D) 2p

Objective : UNDERSTANDING

5. If S = { 1, 2, 3, 4 } , the number of subsets with at least three elements is
   - (A) 3
   - (B) 4
   - (C) 5
   - (D) 15 ( )

6. $(A \cup B') \cap B$ is equal to
   - (A) $A' \cap B'$
   - (B) $A \cap B$
   - (C) $A' \cup B'$
   - (D) A ( )

7. If A = { 1, 2, 3, 4, 5, 6 }
   B = { 1, 3, 5, 7, 9 }
   C = { 2, 4, 6, 8, 10 } ,
   then $n\{ A \cap B \cap C \}$ is equal to
   - (A) 0
   - (B) 1
   - (C) 3
   - (D) 10 ( )

8. In a class of 30 pupils, 12 take needle work, 16 take physics and 18 take history. If all the 30 students take at least one subject and no one takes all three, then the number of puplis taking 2 subjects is

(A) 16
(B) 6
(C) 8
(D) 20 ( )

9. The set $\{ 10, 15, 21, 28 \}$ can be expressed as
(A) $\{ x \mid 10 \leqslant x \leqslant 28 \}$
(B) $\{ x \mid x = \frac{p(p+1)}{2}, p \in N \text{ and } 4 \leqslant p \leqslant 7 \}$
(C) $\{ x \mid 2x + 1 = 21 \}$
(D) none of the above ( )

10. If $A = \{ 1, 2, 3, 6, 8, 10 \}$
$B = \{ 4, 6, 7, 8, 9, 10 \}$
$C = \{ 1, 3, 4, 5, 9, 10 \}$,

then

(A) $A \cup B \cup C = (A \cup B) \cap (A \cup C)$
(B) $(A \cup B) \cap C = (A \cup B) \cap (A \cup C)$
(C) $A \cap (B \cap C) = (A \cup B) \cap (A \cup C)$
(D) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ ( )

Objective : APPLICATION

11. If $A = \{ x \mid x$ is the letters of the word 'follow' $\}$
$B = \{ y \mid y$ is the letters of the word 'wolf' $\}$
and $C = \{ z \mid z$ is the letters of the word 'flow' $\}$,

then

(A) $B = C$ but $A \neq B$
(B) $A = B$ but $B \neq C$
(C) $A = B = C$
(D) A, B, C are different from each other ( )

Objective : SKILL

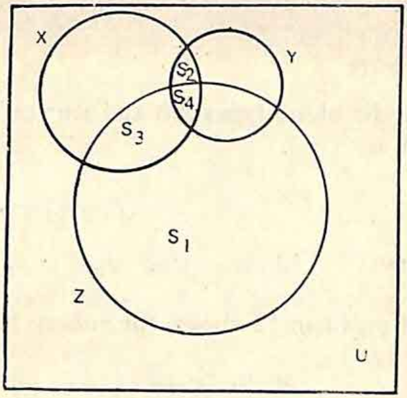


Fig. 1.

12. A commonly accepted means of typing a blood is to match it to any of three types of antigen, A, B and Rh. Every person is classified in two ways. If he has the Rh antigen, he is typed Rh^+, otherwise he is classified Rh^-. He is typed A, B, A B or O, according to which of the other antigens he has; type AB indicates that he has both A and B, and type O that he has neither A nor B.

In the above Venn diagram (fig. 1), U indicates the entire population under study for blood type and define the sets

$X = \{ x \mid x \in U, \text{x having antigen A} \}$

$Y = \{ y \mid y \in U, \text{y having antigen B} \}$

$Z = \{ z \mid z \in U, \text{z having antigen Rh} \}$

The subsets S_1, S_2, S_3 and S_4 are defined as

$S_1 = X' \cap Y' \cap Z$

$S_2 = X \cap Y \cap Z'$

$S_3 = X \cap Y' \cap Z$

and $S_4 = X \cap Y \cap Z$,

where X′, Y′ and Z′ are respectively the complements of X, Y, Z with respect to U.

Then the subset having the blood types A and Rh^+ only is

(A) S_3
(B) S_4
(C) $S_3 \cup S_4$
(D) none of the above ( )

13. The subset having the blood types AB and Rh^- only in the notation of the above problem is
(A) $S_2 \cup S_4$
(B) $S_3 \cup S_4$
(C) S_4
(D) S_2 ( )

14. In the notation of problem 12 above, the subset having the blood types AB and Rh^+ only is
(A) $S_2 \cup S_4$
(B) S_2
(C) S_4
(D) $S_3 \cup S_4$ ( )

15. In the notations of problem 12 above, the subset having the blood types 0 and Rh^+ only is
(A) $S_3 \cup S_1$
(B) $S_4 \cup S_1$
(C) S_1
(D) none of the above ( )

(ii) *True false type* :

In the following statements (No. 1 to 18), write 'T' against each true and 'F' against each false statements at the end of each statement.

Objective : KNOWLEDGE

1. $A \cup B = B \cup A$
2. $A \cap B = B \cup A$
3. $(A \cup B) \cup C = A \cup (B \cup C)$
4. $A \cup \emptyset = \emptyset$

5. $(A \cup B)' = A' \cup B'$

6. $(A \cap B)' = A' \cap B'$

7. $A \cap A' = \emptyset$

8. $A \cap A = A$

9. $(A')' = A'$

Objective : UNDERSTANDING

10. The sets A = { x | x is the letters of the word 'flower' } and B = { x | x is the letters of the word 'flow' } are unequal.

11. The sets A = { x | x is the letters of the word 'wolf' } and B = { x | x is the letters of the word 'follow' } are unequal.

12. The sets { O } and ∅ are the same set.

13. The sets { ∅ } and ∅ are the same sets.

14. The sets { a } and { a, a, a } are equal.

Objective : SKILL

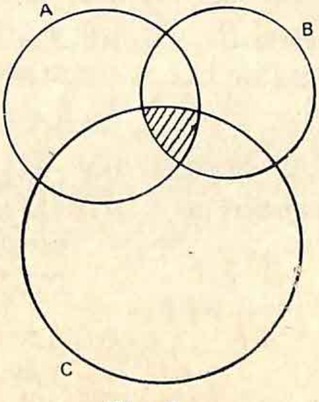


Fig. 2

15. In the above Venn diagram (fig. 2), where A, B, C, are three sets represented by three circles, then the shaded portion represents the set $(A \cap B)\ (A \cap C)$.

16. In the above fig., the shaded portion represents the set $(A \cup B) \cap C$.

17. In the above fig., the shaded region represents the set $[\,(A \cap B) \cap (B \cap C)\,] \cap (A \cap C)$

18. In the above fig., the shaded region represents the set $(A \cap B) \cap C$.

(iii) *Matching type* :

Objective : KNOWLEDGE

1. Match the sets given in column I which are equal to those given in column II in the manner such as (N) — (X) where N denotes an integer and X is an English alphabet.

Column I	*Column II*
(1) $\{1, 2, 3, 4\}$	(A) The set of all the digits in the product of 11 and 25
(2) $\{2, 4, 6, 8,\}$	(B) $\{9\}$
(3) $\{0, 1, 2, 3\}$	(C) $\varnothing$
(4) The set of all odd numbers greater than 7 but less than 11	(D) The set of first four natural numbers
(5) $\{16, 9, 4, 1\}$	(E) $\{5, 6, 7\}$
(6) The set of numbers obtained by adding any pair of numbers in $\{2, 3, 4\}$	(F) $\{1, 2, 3, 0\}$ (G) The set of all the positive integers less than 20, which are perfect squares
(7) All the factors of 10 which are larger than 10	(H) The set of even numbers greater than 1 but less than 10
(8) All the prime factors of 70	

(iv) *Sentence completion type :*

Objective : KNOWLEDGE

1. Fill up the blanks with appropriate answers given in the bracket :

   (A) $A - \emptyset = A$

   (B) $A' - A = \emptyset$

   (C) $(A - B)' = A' \cup B'$

   $(\cup, \cap)$

2. Fill up the blanks with appropriate answers given in the bracket :

   (A) $\{ p \} - \{ p, q, r \}$

   (B) $p - \{ p, q, r \}$

   (C) $\{p, q, r \} - \{ q)$

   $(\in, \supset, \subset)$

Objective : UNDERSTANDING

3. Fill up the boxes in the following diagram (fig. 3) with appropriate numbers given in the bracket.

   $(- \frac{1}{3} , 0 , 1)$

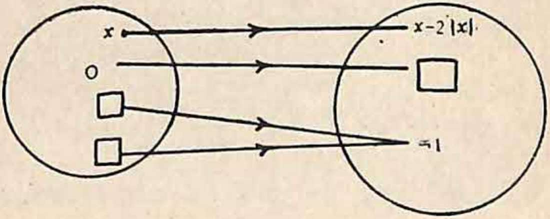


Fig. 3.

Objective : APPLICATION

4. If $A = \{ a, b \}$, $B = \{ 2, 3 \}$ and $C = \{ 3, 5 \}$, then the set $\{ (a, 3) , (b, 3) \}$ is ________

   $[B \times (A \cap C), C \times (B \cap A), A \times (B \cap C)]$

5. Let A = { a, b, c } and B = { x, y, z, w } .
Let the function f consist of a set of ordered pairs (a, x), (b, z), (b, w), (c, w).
Then f : A → B ________________ a mapping.
(forms, does not form)

Objective : SKILL

6.

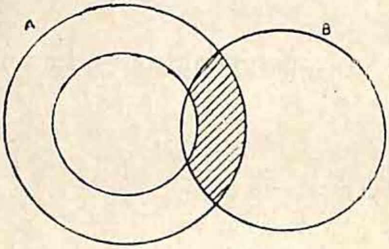


Fig. 4.

In the above Venn diagram (fig. 4) where A, B, C, are three sets represented by three circles and C′ is the complement of C with respect to A, then the shaded portion represents the set ________________
[$(A \cup B) \cap C'$, $(A \cap B) \cap C'$]

CHAPTER VI

UNIT III : EQUATIONS AND INEQUATIONS

FORM : **Essay Type**

Objective : KNOWLEDGE

1. Find the solution of the system of simultaneous equations :
   $x + y + 1 = 0$
   $2x + 3y + 1 = 0$
   and calculate the value of $| x + y | - | 8x - 3y |$.

2. Represent graphically the solution of the following simultaneous inequations :
   (i) $6x + 10y \leqslant 30$
   $10x + 4y \leqslant 20$
   (ii) $x + y \leqslant 9$
   $x \geqslant 3$
   $y \leqslant 7$

3. Represent graphically the solution of the following simultaneous inequations :
   $x + 2y \leqslant 2000$
   $x + y \leqslant 1500$
   $y \leqslant 600$
   $x \geqslant 0$
   $y \geqslant 0$

Objective : UNDERSTANDING

4. Geeta is three times as old as Virendra. After 6 years, Geeta will be 5 times as old as Virendra was 2 years ago. Find their present ages.

5. A relation of the type $y=kx$, where k is a constant, exists between x and y. Some values of x and y are given in the following table :

x	1.2	2.4	3.6	4.8	6.0
y	1.2	0.6	0.9	1.2	1.5

Only one of the entries of y in the above table is incorrect. Find, showing the details, the incorrect entry.

6. If unity is subtracted from the numerator x of a fraction and 2 is added to the denominator y, the fraction becomes 1/3. If unity is added to x and 12 is added to y, the fraction becomes 1/4. Find the fraction.

Objective : APPLICATION

7. Given the table :

x	4	10	6
y	1	--2	0

Find out a relation between x and y of the form $ax + by = c$, where a,b and c are constants. Also find the superfluous data, if any.

Objective : SKILL

8. Solve

$$\frac{a\,(x-a)}{b+c} + \frac{b\,(x-b)}{c+a} + \frac{c\,(x-c)}{a+b} = a+b+c$$

and hence prove that

$$\frac{x}{x-a} = 1 + \frac{a}{b+c} \quad (x \neq a)$$

9. If $P = \{ x, y) \mid x \leqslant 0 \}$ and
$Q = \{ (x, y) \mid 3x + 2y > 7 \}$,
represent P and Q on the co-ordinate plane and shade the region common to both P and Q.

10. The following figure (fig. 5) shows the lines whose equations are $x = 3$, $y = 4$ and $y = 2x + 1$:

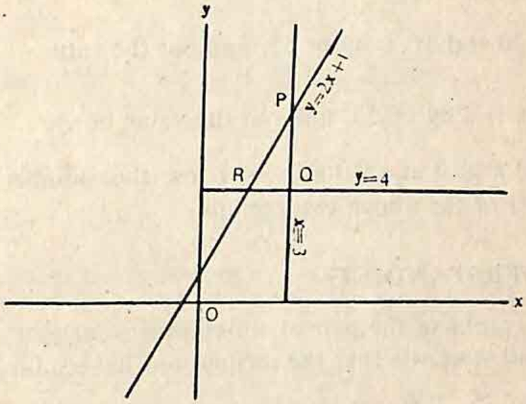


Fig. 5.

(i) Find the co-ordinates of the points P, Q and R.

(ii) Shade the region defined by
$y \geqslant 2x + 1$ and $x \geqslant 3$

11. A playing ground is constructed outside the combined region represented by $x + y \leqslant 1$, $x + y \geqslant 2$, $x - y \geqslant 1$ and $y - x \geqslant 1$ such that the boundary of the ground coincides with the boundary of the above region. A player is running from one corner of the ground to the opposite corner with a uniform speed of v (with the same unit as that of the dimension of the ground) per hour along a straight path. Find the total time taken by the player to reach the opposite corner.

FORM : **Short Answer Type**

Objective : KNOWLEDGE

1. For what value or values of x, the equation
$\dfrac{1}{x-1} = \dfrac{2}{x-2}$ is true ?

2. Find out the solution of the following inequations :
   (i) $3x + 1 > 7$
   (ii) $5x - 8 \geqslant 9 - 12x$

3. If $x + y = 20$ and $3x + 4y = 65$, find out the value of $\frac{x}{y}$ ..

4. If $\frac{x}{y} = 7$, $x + 20y = 54$, find out the value of xy.

5. Represent (i) $x \leqslant 3$ and (ii) $x \geqslant -1$ on the number line. Shade the common part of the above two regions.

Objective : UNDERSTANDING

6. Draw the graphs of the pair of simultaneous inequations in each of the following and state whether the inequations have solution or not.
   (i) $x > 2, x < -2$
   (ii) $x < -3, x > -3$
   (iii) $x - 7 \geqq 0, 5x \leqslant 35$
   (iv) $-5 \leqslant x \leqslant 5, x = 4.5.$

7. Suppose x can take only those values which are elements of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. List the set of values of x which satisfy the the following :
   (a) $x + 2 = 4$
   (b) $x - 1 = 3$
   (c) $x > 6$
   (d) $x < 5$
   (e) $2x < 1$
   (f) $x + 1 = 12$
   (g) $x - 1 > 0$

8. Solve the following equations :
   (i) $(3x + 1)(2x - 4) - x(6x - 7) = 0$
   (ii) $(x + 1)^2 - (x - 1)^2 = 3.$

9. For $0 \leqslant x \leqslant 5$ and $0 \leqslant y \leqslant 5$ and $x, y \in W$ (set of whole numbers), find separately the solution(s) of
   (i) $x + 2y = 8$
   (ii) $x + 2y \leqslant 4$.

10. Find the solution of $2x + 9 > 6$, $x \in R$ and represent it on the number line.

Objective : APPLICATION

11. If $f(x) = px^2 + 7$ and $f(3) = 25$, find the value of p.

12. Find the solution of the following :
   (i) $2x - y > 3$
   and $5x - 2y = 8$
   (ii) $6x + 2y - 10 < 0$
   and $x - y = 3$

Objective : SKILL

13. Indicate the following sets on the number line, where $x \in R$:
   (i) $\{ x \mid x > -2 \}$
   (ii) $\{ x \mid x \leqslant 3 \}$
   (iii) $\{ x \mid x > -2 \} \cap \{ x \mid x \leqslant 3 \}$

14. Represent graphically the solution of $2x - y > 3$
   and $5x - 2y = 8$

15. By drawing the graphs of
   $\{ (x, y) \mid 2x + y \leqslant 3 \}$ and $\{ (x,y) \mid x - y \geqslant 1 \}$,
   shade the region common to both the regions.

16. Represent the following regions graphically :
   (i) $\{ (x,y) \mid y > 0, y < 3 \text{ and } y < 3 - 2x \}$
   (ii) $\{ (x,y) \mid y = 3 - 2x$, x,y are positive integers $\}$.

17. Draw the graph of the equation $y = -2x$ and shade the quadrants through which the line passes.

FORM : **Very Short Answer Type**

Objective : KNOWLEDGE

1. Write the co-ordinates of the point where the lines $x = -1$ and $y = 0$ intersect.
2. In which quadrant does the point $(-4, -3)$ lie?
3. Does the region $x \geqslant -1$ lie on the left or right side of the line $x = -2$?
4. Will the line $4x - 3y + 5 = 0$ pass through the origin ?

Objective : UNDERSTANDING

5. A rectangle is $(3x - 6)$ metres long and x metres broad. If the perimeter of the rectangle is 60 metres, find the breadth of the rectangle.
6. What is the largest positive integer not in the set $\{ x \mid x > 2 \}$?
7. State whether the following systems of simultaneous equations are inconsistent or dependent :

   (i) $2x + y = 3$
   $4x + 2y = 6$

   (ii) $2x + y = 3$
   $4x + 2y = 16.$
8. Express the following statements in the form of an equation or an inequation :
   'In a two-digit number, the sum of two digits is 11 or less than 11'.
9. Express the following statements in the form of an inequation as well as an equation :
   'Arun has 9 books with him. Some of the books are in Bengali and others are in English such that twice the number of Bengali books is always less than the number of English books'.

Objective : APPLICATION

10. Given that p, q and r represents the consecutive even numbers and $p < q < r$. Write down the value of $r - p$.

11. Is it possible to find the solution of the pair of equations obtained by taking one from a pair of dependent equations and another from a pair of inconsistent equations involving two variables x and y?

12. Of all the values of x and y, satisfying both $x + y \geqslant 4$ and $x - y = 0$, what are the least values of inconsistentx and y?

Objective : SKILL

13. Is it correct to say that if $\frac{a}{b} > \frac{c}{d}$, then bc is always greater than ad?

14. Will you get any solution of the following inequations?
    $x + y \leqslant 1.$
    $-0.5x - 5y \leqslant 10,$
    $x, y > 0.$
    If so, what type of solution will you get?

FORM : **Objective Type**

(i) *Multiple choice type* :

In each of the questions (No. 1 to 13), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right-hand margin.

Objective : KNOWLEDGE

1. The correct statement is

   (A) $-3 < \frac{5}{7} < \frac{3}{5} < -\frac{12}{35}$

   (B) $-3 < \frac{3}{5} < \frac{5}{7} < -\frac{12}{35}$

(C) $-3 < -\frac{12}{35} < \frac{3}{5} < \frac{5}{7}$

(D) $-\frac{12}{35} < -3 < \frac{3}{5} < \frac{5}{7}$ ( )

2. If $3x + 1 \geqslant 0, x \in R$, then

(A) $x > -\frac{1}{3}$

(B) $x \leqslant \frac{1}{3}$

(C) $x \leqslant -\frac{1}{3}$

(D) $x \geqslant -\frac{1}{3}$ ( )

3. If $x + y = 7$ and $3x - y = 13$, then the correct solution is
(A) $x = 0, y = 7$
(B) $x = 1, y = 6$
(C) $x = 5, y = 2$
(D) $x = 2, y = -7$ ( )

Objective : UNDERSTANDING

4. If $2x + 3y \leqslant 6$, $y \geqslant 0$, $x \geqslant 0$, then one of the solutions is
(A) $x = -2$
$y = 3$
(B) $x = 1$
$y = 2$
(C) $x = 1$
$y = 1$
(D) $x = 1$
$y = -2$ ( )

5. A linear equation in two variables represents
   (A) a point
   (B) a line
   (C) a circle
   (D) none of the above ( )

6. Two inconsistent linear simultaneous equations will have
   (A) one solution
   (B) two solutions
   (C) no solution
   (D) more than two solutions ( )

7. Two dependent simultaneous linear equations will have
   (A) one solution
   (B) two solutions
   (C) no solution
   (D) more than two solutions ( )

8. A relation between x and y is given by

   $$2y + \frac{x}{3} = \frac{3}{4}$$

   The missing entries x, x, y, the x' s being taken in order, in the table

x	0		
y		0	-1

   are respectively
   (A) $\frac{3}{8}, \frac{3}{8}, \frac{33}{4}$
   (B) $\frac{9}{4}, \frac{33}{4}, \frac{3}{8}$
   (C) $\frac{9}{4}, \frac{3}{8}, \frac{33}{4}$
   (D) none of the above ( )

9. If $x^2 \leqslant 16$, then the statement which is always true is
   (A) $x < 4$
   (B) $x \leqslant 4$
   (C) $x \leqslant -4$
   (D) $-4 \leqslant x \leqslant 4$ ( )

10. If $x^2 \geqslant 16$, then the statement which is always true is
   (A) $x \geqslant 4$
   (B) $x \leqslant -4$
   (C) $x \geqslant 4$ or $x \leqslant -4$
   (D) none of the above ( )

Objective : APPLICATION

11. If a, b, c, d are all non-zero real numbers and if $\frac{a}{b} > \frac{c}{d}$, then the statement $\frac{b}{a} < \frac{d}{c}$ is
   (A) always true
   (B) never true
   (C) true only when $ad > bc$
   (D) true only when two numbers in each of the pairs of numbers a, b and c, d will have the same sign ( )

12. If a, b, c, d are all non-zero real numbers and if $\frac{a}{b} > \frac{c}{d}$, then for the statement $\frac{b}{a} < \frac{d}{c}$ to be always true, it is necessary that
   (A) a, b, c, d should either all be positive or all be negative
   (B) ad should be greater than bc
   (C) two numbers in each of the pairs of numbers a,b and c,d should be of the same sign

(D) two numbers in either of the pairs of numbers a, b and c, d should be positive ( )

Objective : SKILL

13. If $pq = 53.7$ and $p > \sqrt{53.7,}$ then
    (A) $q < \sqrt{53.7}$
    (B) $q = \sqrt{53.7}$
    (C) $q < \sqrt{53.7}$
    (D) none of the above ( )

(ii) *True false type* :

In the following statements (No. 1 to 14), write 'T' against each true and 'F' against each false statements at the end of each statement.

Objective : KNOWLEDGE

1. $2x = 1$, $3x \geqslant 1$ and $x = 3x - 2x$ are respetively an equation, an inequation and an identity.
2. The entire region represented by $x \geqslant -1$ lies on the right side of the y-axis.
3. It is always possible to find the solution of any two linear simultaneous equations involving two variables.
4. By a solution of an equation involving a variable, we mean the value or values of the variable which satisfy the equation.
5. We always get a finite number of solutions of an inequation.

Objective : UNDERSTANDING

6. A pair of dependent equations will always have a finite number of solutions.
7. A pair of dependent equations is always consistent.

8. A pair of consistent equations will have solution which is always finite in number.

9. An equation remains unaltered (as regards the sign of equality and the solution) if we multiply both sides of the equation by any real number. Similarly an inequation also always remains unaltered (as regards the sign of inequality and the solution) if both sides of it are multiplied by any number.

10. An inequation involving two variables can always be solved.

Objective : APPLICATION

11. The regions represented by the inequations $y > |x|$ and $|y| > x$ will be exactly the same.

12. The equations $y = |x|$ and $|y| = x$ are exactly the same.

Objective : SKILL

13. The equation $|x| + |y| = 1$ represents one line only.

14. The region represented by the inequation $x + y \geqslant 1$ includes a point on the circle with origin as the centre, the radius of the circle being $\frac{1}{\sqrt{2}}$

(iii) *Matching type* :

Objective : KNOWLEDGE

1. Match the inequations given in column I with their solutions given in column II in the manner such as (N) — (X), where N denotes as integer and X is an English alphabet.

Column I	*Column II*
(1) $7x + 1 > 0$	(A) $x > 1/3$
(2) $2x - 3 < 0$	(B) $x = 7/8$
(3) $17x > 5x + 4$	(C) $x < 3/2$

(4) $-6x + 4 = -2x$	(D) $x > -1/7$
(5) $13x - 9 = -3x + 5$	(E) $x = 1$

2. In column I, certain inequations are given and the shaded regions represented by these are shown graphically in column II. Match the elements in column I with those in column II in the manner such as (N) – (X), where N denotes an integer and X is an English alphabet.

Column I

(1) $|x| < 2$

(2) $x > 2$ and $x \leqslant -2$

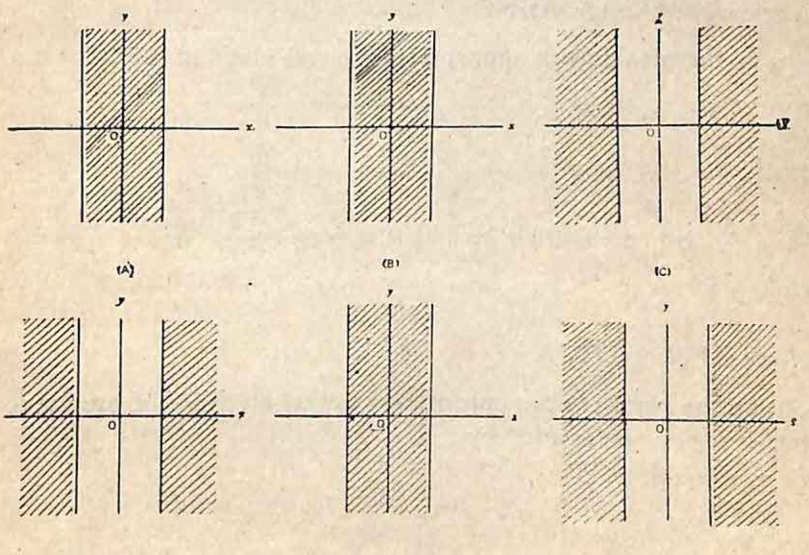
(3) $-2 \leqslant x < 2$

(4) $-2 < x \leqslant 2$

(5) $|x| \geqslant 2$

(6) $x \geqslant 2$ and $x < -2$

Column II



(A) (B) (C) (D) (E) (F)

Fig. 6.

(iv) *Sentence completion type* :

Objective : KNOWLEDGE

1. Fill up the blanks with appropriate answers given in the brackets :

(A) If $x > y$, then $-x$ ______________ y.
($>$, $<$, $=$)

(B) If $x = y$, then $-x$ ______________ y
($>$, $<$, $=$)

(C) The equations $3x - 4y = 1$ and
$-24x + 32y = 59$ are ______________
(dependent, inconsistent)

(D) The point $(-4, 5)$ lies in the ________ quadrant of the co-ordinate planes.
(first, second, third, fourth)

Objective . UNDERSTANDING

2. Fill up the blanks with appropriate answers given in the bracket :

(A) If $0 < x < y$, then $\frac{x}{5}$ ______________ $\frac{y}{4}$

(B) If $0 < x < y$, that $\frac{7}{x}$ ______________ $\frac{3}{y}$

(C) $\frac{3x + 5}{3}$ will be less than $2x + 1$ if x ______________ $\frac{2}{3}$
($<$, $>$)

Objective : APPLICATION

3. Fill up the blanks with appropriate answers given in the brackets :

(A) In the region represented by
$3x + 7y \leqslant 21$,
$x \leqslant 3$,
$y \leqslant 0$,
the line $12x + 7y = 48$ will pass through________point(s) of the region.
(one, two, more than two)

(B) The rectangular region defined by $0 < x \leqslant 1$ and $0 \leqslant y < 2$ will include_____________side (s) of the rectangle.
(one, two, three, four)

Objective : SKILL

4. Fill up the blanks with appropriate answers given in the bracket :

(A) The area of the circle drawn inside the region bounded by $x \leqslant 2$, $x \geqslant -2$ and $0 \leqslant y \leqslant 4$ such that the circle touches all the boundary lines is

____

(π, 4π, 2π)

(B) The area of the region bounded by $0 \leqslant y \leqslant | x |$ and $-2 \leqslant x \leqslant 2$ is

____

(4, 2, 8)

CHAPTER VII

UNIT IV : LOGARITHMS

NOTE : In all problems of this unit, if nothing is mentioned about the bases of log, assume the bases to be 10 unless otherwise mentioned.

FORM : **Essay Type**

Objective : KNOWLEDGE

1. Using the log table, simplify the following :

$$\sqrt{\frac{108.3 \times 5{,}983 \times 3998}{0.00852 \times 19.37}}$$

2. Find the area of the triangle whose sides are 30.05 cm., 20.82 cm. and 40.81 cm. by using the formula

$A = \sqrt{s(s-a)(s-b)(s-c)}$,

where $s = \frac{a+b+c}{2}$, A denotes the area and a, b, c are the sides of the triangle.

Objective : UNDERSTANDING

3. Compute the number of digits in 2^{15} and 3^6.

4. Find the number of zeros after the decimal point in $(\frac{1}{3})^{100}$, given $\log 3 = 0.4771$.

5. Solve the following equations for x and y where $x, y \in N$:

$\log x + \log y = 2$

$x + y = 25$

6. If the logarithm of one-third of the semi-perimeter of a rectangle is half the sum of the logarithm of the length and that of the breadth of the rectangle, then prove that the square of the semi-perimeter of the rectangle is equal to three times the area of the rectangle,

Objective : APPLICATION

7. If $a^2 + b^2 = 7\,ab$, show that
   $\log \{ 1/3\,(a + b) \} = 1/2\,(\log a + \log b)$.

8. Prove that
   (i) antilog x × antilog y = antilog (x + y)
   (ii) antilog (log x) + antilog (log y) = antilog { log (x + y) } (the bases of log and antilog in (ii) being the same).

Objective : SKILL

9. Prove that the equation $\log (x + y) = \log x + \log y$ for positive integral values of x and y holds good only when $x = y = 2$.

10. If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, show that $\left(\frac{21}{20}\right)^{100}$ is greater than 100.

11. Calculate log 0.00598 + log 0.000987 − log 0.000059. Find the antilog of the result thus obtained. Write down an arithmetic expression whose simplified value is this final result.

12. Using log table calculate the [illegible]
   $$2^{.5789^{.9531}} - (2^{.5789})^{.953}$$
   correct upto three places of decimals.

FORM : **Short Aswer Type**

Objective : KNOWLEDGE

1. Find log 0.00015.

2. Express log 48 in terms of log 2 and log 3.

3. Using log table, find the value of $\sqrt[3]{5}$.

4. From the formula $V = \pi r^2 h$, prove that
   $\log r = \frac{1}{2} \{ \log V - \log h - \log \pi \}$

5. Simplify :
   $\log \frac{a}{b} + \log \frac{b}{a} + \log \frac{c}{d} - \log \frac{cy}{dx}$
   and express the result in terms of x and y only.

Objective : UNDERSTANDING

6. Compute the value of log log 1000, given that log 3 = 0.4771.

7. If $2 \log (x + 3) = \log 81$, find x.

8. Without using log table, find the value of $\log 52 - 3 \log 2 - \log 26 + \log 4$.

9. Without using log table, find the value of
   $\log \left(\frac{125 \times 625 \times 2^5}{25} \right)$.

10. If $\frac{\log (1 + x^2)}{\log (2x^2)} = 1$, find x.

11. Solve the following for x :
    $\log x^2 - \log x = 2$.

12. Find the values of a if
    $\log (a^2 - 15a) = 2$.

Objective : APPLICATION

13. If $2 \log | x + 3 | = \log 81$, find x.
    (Compare with problem No. 7 under the heading 'UNDERSTANDING')

14. In the formula, $\log m^n = n \log m$, state the restriction(s), if any, on the signs of m and m^n. Construct an example as an application of the above

formula and prove an absurd result assuming that the above formula does not depend on the signs of m and m^n.

15. Show with two different examples that absurd results can be proved in case we assume that a or b or both a and b in $\log_b a$ are negative.

Objective : SKILL

16. If $\log 2 = 0.3010$, prove that antilog $\bar{1}.0970 = 0.125$ without using log and antilog tables.

17. A student was asked to estimate the value of $(-7)^{-89}$ with the help of a log table. He proceeded as follows :
    Let $x = (-7)^{-89}$
    $\therefore \log x = -89 \log(-7)$ which is not a real number. Finally, he concluded that the given number was not real.
    Do you think that that his conclusion is correct? If not, state (without detailed calculation) how the given number will be calculated with a log table.

Objective : KNOWLEDGE

1. What is the value of $\log a^0$ where $a > 0$?

2. What is the mantissa of log 3 where $\log 3 = 0.4771$?

3. What is the characteristic of an n-digit nmber ?

4. If $\log a = p$ and $\log b = q$, find $\log (a^p b^q)$ in terms of p and q.

Objective : UNDERSTANDING

5. Given that $\log 3 = 0.4771$.
   Find the value of $\log 15 + \log 2$.

6. Express log 15 in terms of $\log \sqrt{15}$.

7. Verify without using a log table:
   $3 \log 2 + 2 \log 5 = 2 + \log 2$.

Objective : APPLICATION

8. What is the number of digits in the integral part of the number log .007 + 4?

9. Given that log 2= .3010. If the length and breadth of a rectangle are respectively log 5 and log 2 (in cm.), what is the perimeter of the rectangle? Is there any superfluous data or statement in the problem? If so, state it.

Objective : SKILL

10. If the length and breadth of a rectangle are respectively (antilog .699) cm. and (antilog .301) cm., state, without using any log table, the area of the rectangle in the simplest form.

FORM : OBJECTIVE TYPE

(i) *Multiple choice type* :

In each of the questions (No. 1 to 9), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right-hand margin.

Objective : KNOWLEDGE

1. If log x = p and log y = q, then log (xy) is equal to
   (A) p^q
   (B) p^q
   (C) q^p
   (D) $p + q$ ( )

2. If antilog p = x, then log x is equal to
   (A) log p
   (B) p

(C) $\frac{1}{\text{antilog p}}$

(D) log (px) ( )

3. log $\frac{\sqrt{ab}}{c^3}$ is equal to

(A) 1/2 log a + log b − 3 log c
(B) 1/2 log a + 1/2 log b − 3 log c
(C) log a + 1/2 log b + 3 log c
(D) 1/2 log a + 1/2 log b − 1/3 log c ( )

Objective : UNDERSTANDING

4. If log x = r, then − r is equal to

(A) log (−x)
(B) log (1/x)
(C) $(\log x)^{-1}$
(D) log (1−x)

5. If log x = s, then 2s is equal to

(A) $\log (x^2)$
(B) $(\log x)^2$
(C) log (2 + x)
(D) log (2x) ( )

6. If log $\sqrt{a}$ = r, then $\sqrt[2r]{a}$ is equal to

(A) $\sqrt{10}$
(B) 100
(C) 10
(D) none of the above ( )

7. If $x = 10^a$, where $a > 0$, satisfies the equation $x^2 - x - 2 = 0$, then a is equal to

(A) 2
(B) log 2

(C) log 1
(D) none of the above ( )

Objective : SKILL

8. Given that antilog $\bar{1}.8445 = .6990 = \log 5$. Then $(\log 5)^{\log 5}$ is equal to
   (A) antilog [−. 6990 × .1555]
   (B) −. 6990 × .1555
   (C) − antilog .6990 × antilog .1555
   (D) $\bar{1}.8445 \times .6990$ ( )

9. Given that antilog $\bar{1}.8445 = .6990 = \log 5$. Then .6990 × antilog $\bar{1}.8445$ is the value of
   (A) $(\log 5)^{\log 5}$
   (B) $\log 5^{\log 5}$
   (C) log 5 × log 5
   (D) log 5 × log (log 5) ( )

(*ii*) *True false type* :

In the following statements marked with 1, 2, 3, etc., write 'T' against each true and 'F' against each false statements at the end of each statement.

Objective : KNOWLEDGE

1. $\log_a 1 = 0$, $a = 0$.
2. If log (25.36) = 1.4041, then antilog (0.4041) is 0.2536.
3. If $\log_a b = m$, then $b^m = a$.
4. The mantissa of log 19.5734 is equal to the mantissa of log 195.734.
5. The characteristic of log 35.725 is equal to the characteristic of log 357.256.
6. $1/2 \log \frac{\sqrt{ab}}{c} = 1/4 \log a + 1/4 \log b + 1/2 \log c$.
7. $\log\frac{a}{b} + \log\frac{b}{c} + \log\frac{c}{a} = 0$.
8. $\log a + \log b + \log c = 0 \Rightarrow abc = 0$.

Objective : UNDERSTANDING

9. If $a < 0$ and $b < 0$, then $\log (ab) = \log | a | + \log | b |$.

10. If $abc = 1$, then $\log a^k + \log b^k + \log c^k = k$.

11. $2/3 > 0 \Rightarrow \log 2/3 > 0$.

Objective : APPLICATION

12. If $y^{\frac{1}{q}} = z^{\frac{1}{r}}$ where q and r are known quantities and $\frac{x}{y} = \frac{y}{z} = 2$.
then $\log x + \log y + \log z$ can be expressed in terms of the known quantities.

Objective : SKILL

13. The value of
$\log \{ \text{antilog} (-.48) \times \text{antilog} (-.52) \}$ is -1.

(iii) *Matching type* :

Objective : KNOWLEDGE

1. In column I, log and antilog of some numbers are given and in column II, their values are given. Given that $\log 3 = 0.4771$, match those given in column I with those given in column II in the manner such as (N) — (X), where N denotes a positive integer and X is an English alphabet.

Column I	*Column II*
(1) log .03	(A) .3
(2) log .3	(B) $\bar{2}.4771$
(3) log 300	(C) .0003
(4) antilog $\bar{4}.4771$	(D) $\bar{1}.4771$
(5) antilog 3.4771	(E) 2.4771
(6) antilog $\bar{1}.4771$	(F) 3000

2. The logarithm of certain numbers are given in column I and their values are given in column II. Without using any log table, match these ele-

ments in the manner such as (N) — (X), where N denotes a positive integer and X is an English alphabet.

Column I	*Column II*
(1) $\log_3 9$	(A) 3
(2) $\log_4 64$	(B) 2
(3) $\log_{10} 1$	(C) 1
(4) $\log_3 3$	(D) 0

3. Certain numbers are given in column I and the characteristics of the logarithms of those numbers are given in column II. Without using a log table match these elements in the manner such as (N) — (X), where N denotes a positive integer and X is an English alphabet.

Column I	*Column II*
(1) 431.6	(A) 5
(2) .0000832	(B) 0
(3) .7373	(C) $\bar{1}$
(4) 763900	(D) $\bar{5}$
(5) $.0063 \times 10^3$	(E) 6
(6) 6315×10^{-7}	(F) 2
(7) $10^5 \times 10^3 \times 10^{-2}$	(G) $\bar{4}$

4. Certain numbers are given in column I and their antilogs are given in column II. Use log table to match these elements in the manner such as (N) — (X), where N deotes a positive integer and X denotes an English alphabet.

Column I	*Column II*
(1) 1.2465	(A) 64.8
(2) $\bar{2}.8946$	(B) .00619
(3) $\bar{3}.7917$	(C) 61900

(4) 4.7917	(D) 17.64
(5) 1.8116	(E) .07846

5. Logarithms of some numbers are given in column I and their values are given in column II. It is given that $\log 3 = .4771$ and $\log 4 = .6021$. Without using any log table, match the elements in colums I and II in the manner such as (N) − (X), where N denotes a positive integer and X is an English alphabet.

Column I	*Column II*
(1) $\log .003$	(A) $\bar{1}.1250$
(2) $\log .003 - \log .0004$	(B) $\bar{1}.8750$
(3) $\log .0004 - \log .003$	(C) -2.5229
(4) $\log .75$	(D) $.8750$

(iv) *Sentence completion type* :

Objective : KNOWLEDGE

1. Fill up the blanks with appropriate answers given in brackets :

(A) The characteristic of log 3.729 is _______________.
(0, 3, 3729)

(B) $\log (m^n)^p =$ __________ × log m.
(n^p, np)

(C) $\log m^{n^p} =$ __________ × log m.
(n^p, np)

(D) $a > 0$, $b > 0$ and $\log (ab) = 0 \Rightarrow$ _______________.
($ab = 0$, $a + b = 0$, $a = \frac{1}{b}$, $a + b = 1$)

Objective : UNDERSTANDING

2. Fill up the blanks with the appropriate answers given in the brackets:

(A) If x is the logarithm of of a to the base b, then it implies that_____.
($a^x = b$, $b^x = a$, $x^a = b$, $x^b = a$, $a^b = x$, $b^a = x$)

(B) In the formula $\log m^n = n \log m$, it is necessary that__________.
($m^n > 0, m > 0, n > 0$)

Objective : APPLICATION

3. Fill up the blank with appropriate answer given in the bracket in the following :
We denote the expression :
log log log·· ·········log x (where there are n log's) by the symbol $1^{(n)}$ x.
If $1^{(n)}(x) = 0$ for $x = 10^{10^{10}}$, then the value of n is______________.
(10, 110, 3, 4)

Objective : SKILL

4. Fill up the blank with appropriate answer given in the bracket :
If $\log 100 = 2$ and $\log 50 = 1.6990$, then log 8 is______________.
(.9030, .3090, .9300, .3900)

(*v*) *Only wrong answer type* :

In each of the questions (No. 1 to 5), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is **incorrect** and others are **correct.** The letter that has been given against the **incorrect** answer should be written in bold letter in the bracket provided in the right-hand margin.

Objective : KNOWLEDGE

1. (A) $\log 2^2 = 2 \log 2$
   (B) $\log a^2 = 2 \log a$
   (C) $\log 1^2 = 2$
   (D) $\log 5^2 = 2 \log 5$ ( )

2. The value of log 1000 is
   (A) 3
   (B) $\log 10^2 + 1$
   (C) $3 \log 10$

Case II

March 6	By balance			Rs 1100
March 10	By cash		Rs 300	Rs 1400
March 26	To cheque	Rs 900		Rs 500

21. Surendra owns a house worth Rs 100000. He sells it to Virendra at a profit of 10%. Virendra sells the house back to Surendra at a loss of 10%. How much does Surendra make out of this ?

22. The price of an article is cut by 10%. By what per cent must the new price be increased to restore it to its former value ?

23. Find the difference between a discount of 40% on Rs 10000 and two successive discounts of 36% and 4% on the same amount.

24. By selling an article for Rs 1260, a man gains by 5%. What should be the selling price if he wants to gain 7% ?

Objective : APPLICATION

25. Use the appropriate formula to find the population of a city after 2 years, if it increases at the rate of 10% per annum when the present population is 10000.

26. A reduction of 20% in the price of sugar enables me to get 15 kg more for Rs 240.00. What is the reduced price ?

Objective : SKILL

27. Find the least number of complete years in which a sum of money put at 20% compound interest will be more than doubled.

FORM : **Very Short Answer Type**

Objective : KNOWLEDGE

1. A machine is sold for Rs 85 after getting a profit of Rs 5 on it. What is the percentage of profit ?

2. A T.V. set is bought for Rs 2400 and sold at a gain of 5%. Find its selling price.

3. Find the annual income obtained from 50 shares of Rs 15 each, if the dividend is declared at $3\frac{1}{2}\%$.

Objective : UNDERSTANDING

4. Some money is borrowed at the rate of $6\frac{1}{4}\%$ simple interest per annum. In how many years is the principal doubled ?

5. In the formula for compound interest $A = P\,(1+0.02r)^n$, find the rate of interest.

Objective : APPLICATION

6. If principal = P in rupees, time = n in years, rate of compound interest = r%, write the formula to calculate the amount received after n years, if the interest is compounded quarterly.

Objective : SKILL

7. $\frac{1}{3}$rd of the profit of Meera is equal to $\frac{5}{9}$th of the loss of Mohan. If the loss of Mohan is equal to Rs 2, express the loss of Mohan as a percentage of the profit of Meera.

FORM : **Objective Type**

(i) *Multiple choice type* :

In each of the questions (No. 1 to 7), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right hand margin.

Objective : KNOWLEDGE

1. If C.P. = Rs 80.00, overhead = Rs 20.00 and S.P. = Rs 120.00, the profit is

(A) 20%
(B) 50%
(C) Rs 40.00%
(D) 40% ( )

2. The annual divident on a share of 5% stock (par value Rs. 50.00) is
(A) Rs 5.00
(B) Rs 2.50
(C) 2½%
(D) Rs 10.00 ( )

3. Adding x% of y to y is the same as multiplying y by
(A) $\frac{x}{100}$
(B) $x\left(\frac{1}{100} + 1\right)$
(C) $\frac{x}{100} + 1$
(D) none of the above ( )

Objective : UNDERSTANDING

4. A house and a car were sold for Rs 12000.00 each. The house was sold at a loss of 20% of the cost and the car at again of 20% of the cost. The entire transaction results in
(A) no loss or gain
(B) loss of Rs 1000.00
(C) gain of Rs 1000.00
(D) none of the above ( )

5. A rising tradesman adds each year an amount equal to 10% of his capital to his capital at the end of the year. If on Ist January, 1975 his capital was Rs 10000.00, then on January 1st, 1978, his capital was
(A) Rs 13000.00
(B) Rs 13310.00
(C) Rs 14641.00
(D) none of the above ( )

Objective : APPLICATION

6. A sum of money becomes eight fold in 3 years at compound interest rate which is
   (A) 100%
   (B) 1%
   (C) 8%
   (D) none of the above ( )

Objective : SKILL

7. A sum of money placed at compound interest doubles itself in 4 years. It will amount to eight times itself in
   (A) 16 years
   (B) 8 years
   (C) 12 years
   (D) none of the above ( )

(ii) *True false type* :

In the following statements (No. 1 to 13) marked with 1, 2, 3, etc., write "T" against each true statement and "F" against each false statement at the end of each statement.

Objective : KNOWLEDGE

1. 8% of the total capital stock of Rs 70000.00 is Rs 5600.00.
2. $\frac{7}{16}$ can be expressed as $23\frac{3}{4}\%$.
3. x% of y equals x $(\frac{y}{100})$.
4. The compound interest on Re 1.00 at 6% per annum for 4 years is Rs $(1.06)^4$.
5. 300 Kg. of rice bought at Rs 2.80 per Kg. and sold at Rs 280.00 per quintal gives a profit of 3%.

(D) $2 \log 500$ ( )

3. (A) $\log m^n = n \log m$
   (B) $\log (mn) = \log m + \log n$
   (C) $\log \frac{m}{n} = \log m - \log n$
   (D) $\log (m - n) = \frac{\log m}{\log n}$ ( )

Objective : UNDERSTANDING

4. (A) $4 \log 2 < 1 < 3 \log 2$
   (B) $\log 16 > \log 10 > \log 8$
   (C) $4 \log 2 > 1 > 3 \log 2$
   (D) $2 \log 4 > \log 2 + \log 5 > \log 8$ ( )

Objective : APPLICATION

5. (A) $\log x > 0 \Rightarrow x > 1$
   (B) $\log x = 10 \Rightarrow x = 100$
   (C) $\log x = 0 \Rightarrow x = 1$
   (D) $\log x < 0 \Rightarrow 0 < x < 1$ ( )

CHAPTER VIII

UNIT V : COMMERCIAL MATHEMATICS

FORM : **Essay Type**

Objective : KNOWLEDGE

1. Find the difference between the simple and compound interests on Rs 1250 for two years at 4 per cent per annum ?
2. Prove that the amount at compound interest for 2 years at $2\frac{1}{2}$ percent is 1.050625 times the principal.
3. A shop was bought for Rs 10000. For how much should it be sold to gain 15% ?

Objective : UNDERSTANDING

4. On what sum will the difference between the simple and compound interests for 3 years at 5% per annum amount to Rs 36.60 ?

Objective : APPLICATION

5. A dishonest tradesman uses false weights in buying goods from a wholesaler as well as in selling it to a customer and gains 10% each time. If the sale price be the same as the cost price, how much per cent does he gain by his dishonesty ?
6. How much per cent must a tradesman add on the cost price of his goods so that he may make 20% profit after allowing his customers a discount of 10% on his bill ?
7. The state bank pays 6% simple interest per annum. A businessman deposits Rs 20000 at the beginning of a year and after 3 months he

withdraws Rs 12000. After 2 months from the date of withdrawal of the amount, he again deposits Rs 8000. Three months before the closing of the year, he further deposits Rs 4000. Compute the total interest for the year.

8. A man borrows some money on compound interest and returns it in two years in equal annual instalments. If the rate of interest is 5% and yearly instalment is Rs 441, find the principal.

Objective : SKILL

9. A boy got 43 marks in a paper and heard that had he got 2 marks less, he would have just failed in that paper. If 30% is the pass mark and the fractional mark is not allowed, then what is the full marks in that paper ?
10. A sells his goods 25% cheaper than B and 25% dearer than C. A customer buys Rs 750 worth of goods from A. Would it be profitable for him to have bought half the goods from B and half the goods from C ? Calculate the profit or loss of the customer.

FORM : **Short Answer Type**

Objective : KNOWLEDGE

11. An article which costs Rs 75 was sold at a loss of 5%. What is the selling price of the article ?
12. Find the interest on Rs 5000 at 4% from April 7, 1978 to August 30, 1978. (Both April 7 and August 30, 1978 are to be included in the calculation of interest).
13. What Principal will amount to Rs 912 in 4 years at $3\frac{1}{2}$% simple interest per annum ?
14. In what time will Rs 1250 amount to Rs 1375 at 5% simple interest per annum ?
15. A refrigerator is sold for Rs 4500 less successive discounts of 20% and 15%. What is the sale price of the refrigerator ?

16. 6 pens are bought at Rs 5 and 5 pens are sold at Rs 6. What is the gain or loss in per cent ?

Objective : UNDERSTANDING

17. A merchant buys 3 pens for Rs 10 and an equal number at 5 for Rs 20. At what rate must he sell the pens in order to have no gain or loss.

18. The balance in Shakeel's account in a bank was Rs 500 on the 1st January of a year. He deposited Rs 300, Rs 2000 and Rs 2400 on January 7, January 10 and January 12 respectively and he withdrew Rs 300 and Rs 2400 on January 4 and January 11 respectively. If the rate of interest be 6% per annum and if the interest is paid on the minimum balance from the 10th to the end of the month, then calculate shakeel's interest for the month of January.

19. Calculate the interest in the following pass book from January to March at 5% per annum :

Date	*Particulars*	*Dr.*	*Cr.*	*Balance*
Jan 1	By balance			Rs 10
Jan 20	By cash		Rs 600	Rs 610
Feb 16	By cash		Rs 390	Rs 1000
Feb 26	To cheque	Rs 300		Rs 700
March 6	By cheque		Rs 400	Rs 1100
March 10	To cheque	Rs 900		Rs 200
March 26	By cheque		Rs 300	Rs 500

20. Calculate the interest at the rate of $4\frac{1}{2}\%$ in the following two cases. Which of these two will give more interest ?

Case I

Date	*Particulars*	*Dr.*	*Cr.*	*Balance*
March 6	By balance			Rs 1100
March 10	To cheque	Rs 900		Rs 200
March 26	By cash		Rs 300	Rs 500

Objective : APPLICATION

6. If $1 + \cos^2\theta = \frac{1}{\cos^2\theta}$ $(0 \leqslant \theta \leqslant 90^\circ)$, prove that $\sin\theta + \sin^2\theta = 1$.

7. If the angle θ is acute and if
$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{1}{\tan\theta}$, then find the value of θ.

8. Prove that
$$\frac{1 + 3\sin\theta - 4\sin^3\theta}{1 - \sin\theta} = (1 + 2\sin\theta)^2.$$

9. If $\frac{1 + \sin\theta}{1 - \sin\theta} = m^2$, then find the value of $\tan\theta + \frac{1}{\cos\theta}$.

10. On one side of a river, there is a vertical tree and there is an observer on the other side of the river. The observer wishes to measure the height of the tree without crossing the river, since it is difficult to cross it. So he chooses two points A and B on that side of the river where he stands such that the foot of the tree lies on AB. He measures the angles of elevation of the top of the tree from A and B with the help of an instrument as well as the distance AB. If the angles of elevation from A and B be 45° and 30° respectively and if AB = 4 metres, find the height of the tree.

Objective : SKILL

11. The angles of elevation of the bottom and top of a flag staff at the top of a building as seen from a point at a horizontal distance of 10 metres from the foot of the building are 30° and 45° respectively. Find the height of the flag staff.

12. Using the result
$\sin(A - B) = \sin A \cos B - \cos A \sin B$, compute the value of $\sin 15^\circ$.

13. If $\frac{\sin^4 A}{\cos^2 B} + \frac{\cos^4 A}{\sin^2 B} = 1$,
prove that $\frac{\cos^4 B}{\sin^2 A} + \frac{\sin^4 B}{\cos^2 A} = 1$.

FORM : Short Answer Type

Objective : KNOWLEDGE

1. If $A = 60°$ and $B = 30°$, verify that $\sin (A + B) = \sin A \cos B + \cos A \sin B$.
2. If $\theta = 90°$, verify the following result :
$$\cos \frac{\theta}{2} = \tfrac{1}{2} \sqrt{1 + \sin \theta} + \tfrac{1}{2}\sqrt{1 - \sin \theta}.$$
3. Find the value of $8 \sin 30° \sin 60° \sin 90°$.
4. Show that $(\cos \theta + \sin \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$.

Objective : UNDERSTANDING

5. If $\sin \theta = \dfrac{m^2 - n^2}{m^2 + n^2}$ $(m > n)$, find $\cos \theta$ in the simplest form.
6. Prove that $\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta$.
7. Express $1 - 2 \sin \alpha \cos \alpha$ as a perfect square.
8. Show that $\sin^5 \theta \cos^3 \theta + \cos^5 \theta \sin^3 \theta = \sin^3 \theta \cos^3 \theta$.
9. Prove the following identity :
$$\frac{2 \cos^2 \theta - \sin^2 \theta + 1}{\cos \theta} = 3 \cos \theta.$$
10. A ladder leaning against a vertical wall makes an angle of 60° with the ground. The foot of the ladder is 4 metres from the wall. Determine the length of the ladder.
11. If $\tan x = \dfrac{1}{\sqrt{3}}$ and $\sin y = \dfrac{1}{\sqrt{2}}$, where x and y are acute angles, show that $x + y$ is also an acute angle.

Objective : APPLICATION

12. If $\sin A + \cos A = 1$, what is the value of $\sin A - \cos A$.

13. Assuming that $\sin(A+B) = \sin A \cos B + \cos A \sin B$, deduce that $\sin^2 A + \cos^2 A = 1$.

14. If $x = \frac{\cos A}{1 - \sin A}$, then show that $\frac{1}{x} = \frac{\cos A}{1 + \sin A}$.

15. If x and y are complementary angles, then show that $\sin x \cos y = 1 - \cos^2 x$.

16. If a and b are two real numbers, prove that $\sin\theta = a + \frac{1}{a}$ is impossible.

Objective : SKILL

17. If $\tan\theta = \frac{3}{4}$, find the value of $\cos^2\theta - \sin^2\theta$.

18. If $\tan\theta = \frac{m}{1}$, find the value of $\frac{a\cos\theta + b\sin\theta}{a\cos\theta - b\sin\theta}$.

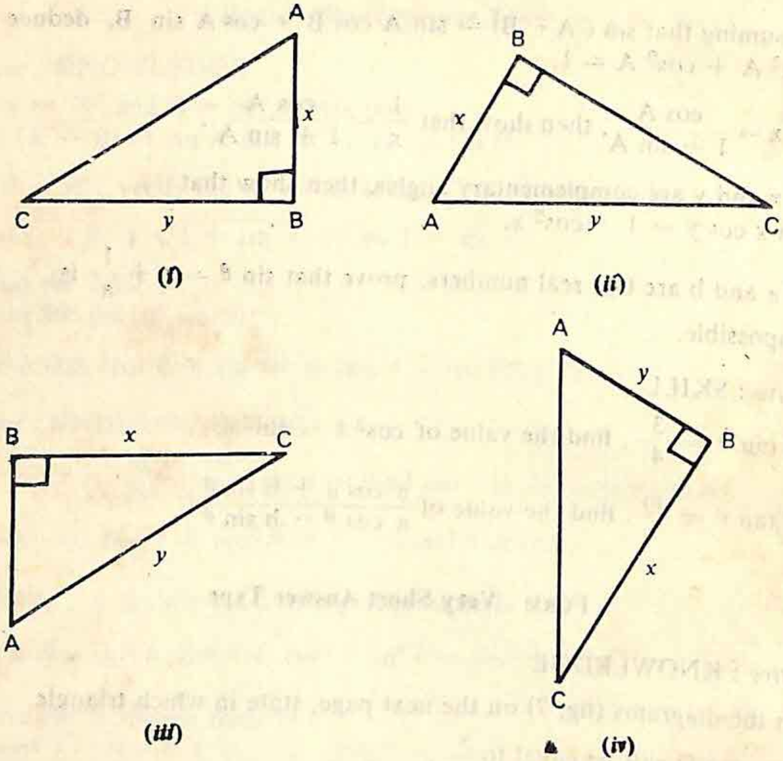
FORM : **Very Short Answer Type**

Objective : KNOWLEDGE

1. In the diagrams (fig. 7) on the next page, state in which triangle
   - (a) tan C will be equal to $\frac{x}{y}$.
   - (b) sin A will be equal to $\frac{x}{y}$.
   - (c) tan A will be equal to $\frac{x}{y}$.
   - (d) cos A will be equal to $\frac{x}{y}$.

2. Given $\sin 18^\circ = \frac{\sqrt{5} - 1}{4}$, what is the value of $\cos 72^\circ$?

3. Given $\cos 36^\circ = \frac{\sqrt{5} + 1}{4}$. Write down the value of $\sin 54^\circ$.



(i) (ii) (iii) (iv)

Fig. 7.

4. State the angle for which $\tan \theta$ is not defined.

Objective : UNDERSTANDING

5. Choose suitable values of θ and ϕ and verify that $\sin (\theta - \phi) \neq \sin \theta - \sin \emptyset$.

6. Write an acute angle for which $\sin \phi = \cos \phi$.

6. If the rate of increase of the value of a certain article is (2r)%, then the value of the article will amount to $(1 + \frac{r}{50})^n$ times its C.P. at the end of n years.

7. If the rate of compound interest is 10% per annum compounded annually, a certain sum of money will become exactly double of itself in 10 years.

8. If the par value of a share is p, the number of shares is n and r% be the rate of annual dividend, then the annual income is $\frac{pnr}{100}$

Objective : UNDERSTANDING

9. M is greater than N by (M−N)%.

10. For the same amount, the simple interest and the compound interest compounded annually at the same rate for one year will be the same.

11. Successive discounts of 20% and 5% are equal to a single discount of 24%.

12. The discount on Rs 3000.00 on the term 2/20 is Rs 60.00 if paid on the 10th day.

13. If the difference in the simple interest and the compound interest on the same sum of money at 10% for 2 years is Re 1.00, then the sum is Rs 121.00.

(iii) *Sentence completion type* :

Fill up the blanks with appropriate answers given in the brackets :

Objective : KNOWLEDGE

1. 5% of 5% of Rs 100.00 is____________________________.
   (Rs 25.00 Rs 10.00, Rs 0.25)

2. If Rs 100.00 amount to Rs 121.00 in 2 years, the rate of compound interest calculated annually is ____________________________ .
   (21%, 10%, 10½%)

3. Successive discounts of 10% and 20% are equivalent to a single discount of____________________.
(30%, 28%)

4. If y be the principal and x% be the rate of simple interest in 10 years, then the total interest will be ____________________.
(0.1 xy, 0.01 xy, xy)

5. If the list price be y and the rate of discount be 100x%, then the price after the discount will be____________________.
(y−x, y−xy)

6. The decimal representation of 5% is____________________.
(0.5, 0.05, 0.005)

7. To calculate the amount received after 2 years at 5% compound interest annually, we multiply the principal by ____________________.
($.05^2$, 1.5^2, 1.05^2)

Objective : UNDERSTANDING

8. If n be the number of years, r% be the rate of interest and the simple interest $= \frac{P\,n\,r}{50}$, then the principal is____________________.
(P, $2\,P$, $\frac{P}{2}$)

9. In the formula
$P = a\,(1 + \frac{r}{100})^n$, the principal is____________________.
(P, a)

10. In the formula
$a = (1 + \frac{r}{100})^n$, the principal is____________________.
(a, 0, 1)

11. In the formula

$2P = P(1 + \frac{r}{100})^n$, the compound interest is ____________________.

(P, 2P, unknown)

Objective : APPLICATION

12. The income derived on investing Rs 1188.00 in 3% stock at 81 is

____________________.

(Rs 35.64, Rs 44.00, unknown)

13. If A earns 25% more than B, then the per cent by which B earns less than A is ____________________.

(20%, 25%)

CHAPTER IX

UNIT VI : TRIGONOMETRIC RATIOS AND THEIR APPLICATIONS

FORM : **Essay Type**

Objective : KNOWLEDGE

1. Let ABCD be a square of side 8 cm. Let M be the mid-point of AB. If $\angle$ BMC = θ (in degrees), find $\sin\theta \cos\theta$ and $\tan\theta$.

2. Prove that

$$\left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2 = \frac{1+\cos\theta}{1-\cos\theta}.$$

3. Prove that

$$\frac{1}{1-\sin^2\theta} - \frac{1}{\frac{1}{\sin\theta^2}-1} = 1$$

Objective : UNDERSTANDING

4. If $\sin (A + B = \frac{\sqrt{3}}{2}$ and $\cos (A-B) = \frac{1}{\sqrt{2}}$, find A and B where A + B and A − B are acute angles

5. If x, y, z are three real numbers, explain that $\sin\theta = \frac{x^2+y^2+z^2}{xy+yz+xz}$ is impossible.

Objective : APPLICATION

7. What is the value of $\sin 18^{\circ} \cos 72^{\circ} + \sin^2 72^{\circ}$?

8. Can $\sin \theta + \cos \theta$ have a minimum value 0 when $0 \leqslant \theta \leqslant \frac{1}{2}$? If so, state the value of θ for which the value of $\sin \theta + \cos \theta$ is zero.

9. Can $\sin \theta$, $\cos \theta$ $(0^{\circ} \leqslant \theta \leqslant 90^{\circ})$ and $\sqrt{2}$ be the sides of a triangle ? If not, state the reason.

10. Is it possible to construct a triangle with sides of magnitudes $\sin^2 \theta$, $\cos^2\theta$ and 1 ? If not state the reason.

Objective : SKILL

11. Let ABC be a triangle such that $\angle A = 45^{\circ}$ and $\angle C = 60^{\circ}$. From B, draw a perpendicular BD on AC. If AD = 6 cm, find DC.

12. ABC is a right-angled triangle and $\angle B$ is a right angle. E and D are respectively the points BC on and BA produced such that $AC \parallel DE$. If $\angle DEB = 60^{\circ}$, find the ratio $\frac{BC}{AB}$.

FORM : **Objective Type**

(i) *Multiple choice type*

In each of the questions (No. 1 to 7), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right hand margin.

Objective : KNOWLEDGE

1. $\cos (90^{\circ} - \theta)$ is equal to
   (A) $-\sin \theta$
   (B) $\sin \theta$
   (C) $\cos \theta$
   (D) none of the above ( )

2. $1 - \cos^2 60°$ is equal to
   (A) $\cos^2 30°$
   (B) $\sin 60°$
   (C) $-\sin^2 60°$
   (D) none of the above ( )

Objective : UNDERSTANDING

3. The greatest value of $\sin \theta + \cos \theta$ $(0° \leqslant \theta \leqslant 90°)$ is
   (A) $\sqrt{2}$
   (B) $\frac{1}{\sqrt{2}}$
   (C) 1
   (D) 2 ( )
4. $\sin^4 \theta + \cos^4 \theta$ is the same as
   (A) $\sin^2 \theta^2 + \cos^2 \theta^2$
   (B) $\sin \theta^4 + \cos \theta^4$
   (C) $(\sin \theta + \cos \theta)^4$
   (D) none of the above ( )
5. $\cos^2 6° + \sin^2 84°$ is equal to
   (A) $\cos (6°)^2 + \sin (84°)^2$
   (B) $\cos^2 (6° + 6°)$
   (C) $2 \sin^2 84°$
   (D) none of the above ( )

Objective : APPLICATION

6. If $\tan \theta = \frac{1}{3}$ then
   (A) $\sin \theta = 1$ and $\cos \theta = 3$, since $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{3}$
   (B) $\sin \theta$ and $\cos \theta$ cannot be uniquely determined
   (C) $\sin \theta = \frac{1}{4}$ and $\cos \theta = \frac{3}{4}$ since
   $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{3}$
   (D) $\sin \theta$ and $\cos \theta$ have values other than the above ( )

Objective : SKILL

7. ABC is a right-angled triangle with $\angle$ B as a right angle and BC as a base. If sin $\angle ACB = \frac{1}{2}$ then
   (A) we can draw a unique triangle
   (B) we can draw only two triangles
   (C) we can draw more than two equiangular triangles
   (D) we can not draw any triangle at all with the given results ( )

(ii) *True false type*

In the following statements marked with 1,2,3, etc., write "T" against each true statement and "F" against each false statement.

Objective : KNOWLEDGE

1. If a is a real positive number and θ is acute, then $\sin\theta = 1 + a$ must hold good.
2. $\frac{\sin 45^\circ}{\cos 45^\circ} = \frac{1}{\tan 45^\circ}$.
3. $(1 - \cos A)(1 + \cos A) = \sin^2 A$.
4. $\sin 60^\circ + \sin 30^\circ = \sin 90^\circ$.
5. $\sin 2^\circ$ and $\cos 2^\circ$ are not defined.

Objective : UNDERSTANDING

6. If $\sin\theta = \frac{3}{4}$ then $\tan\theta = \frac{5}{4}$ and $\cos\theta = \frac{3}{5}$

   since $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{4}}{\frac{3}{5}} = \frac{5}{4}$
7. $\sin 24^\circ 6' 30'' = \cos 65^\circ 53' 30''$.
8. $\cos 75^\circ = \cos 15^\circ$.

9. If the angle of elevation of a vertical pole is 45°, then the height of the pole is always equal to the horizontal distance from the observer to the bottom of the pole.

10. $\cos^2 \theta + \cos^2 (90° - \theta)$ is constant for all values of θ.

11. If θ lies in the first quadrant, the value of $\sin \theta$ decreases as θ increases.

12. cosine of an angle is the same as the sine of its complementary angle.

Objective : APPLICATION

13. If $0 < \cos \theta < \frac{1}{2}$, then $\sin \theta > \frac{1.5}{\sqrt{3}}$.

Objective : SKILL

14. If $\sin \theta$ is 0.7, then θ will be greater than 45°.

(iii) *Matching type* :

Objective : KNOWLEDGE

1. In column I, the trigonometric ratios are given and in column II, their values are given with reference to the figure given below. Match the elements of column I with those in column II in the manner like (N) – (X), where N denotes a positive integer and X is an English alphabet.

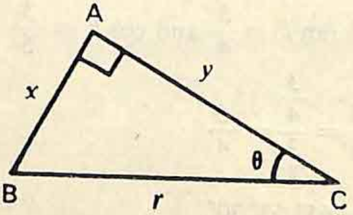


Fig. 8.

Column I	*Column II*
(1) $\sin\theta$	(A) $\frac{y}{x}$
(2) $\cos\theta$	(B) $\frac{x}{r}$
(3) $\tan\theta$	(C) $\frac{y}{r}$
(4) $\tan(90^\circ - \theta)$	(D) $\frac{x}{y}$

Objective : UNDERSTANDING

2. Match the elements of column I, which are equal to those in column II in the manner like (N) — (X), where N is a positive integer and X is an English alphabet.

Column I	*Column II*
(1) $\frac{\sin 80^\circ}{\cos 10^\circ}$	(A) $\sin 36^\circ$
(2) $\cos 54^\circ$	(B) 1
(3) $\sin 54^\circ$	(C) $\frac{\cos 15^\circ}{\sin 15^\circ}$
(4) $\tan 75^\circ$	(D) $\cos 36^\circ$

3. In column I, ranges of values of θ are given for which the statement in column II will be true. Match those given in column I with those given in column II in the manner like (N) — (X), where N is a positive integer and X is an English alphabet.

Column I	*Column II*
(1) $0^\circ < \theta < 45^\circ$	(A) $\tan\theta \geqslant 1$
(2) $0^\circ < \theta < 90^\circ$	(B) $0 < \cos\theta \leqslant 1$
(3) $0^\circ \leqslant \theta < 90^\circ$	(C) $\tan\theta > 0$
(4) $45^\circ \leqslant \theta < 90^\circ$	(D) $0 < \sin\theta < \frac{\sqrt{2}}{2}$

(*iv*) *Sentence completion type*

Fill up the blanks with appropriate answers given in the brackets :

Objective : KNOWLEDGE

1. In a right-angled triangle ABC, the hypotenuse AB = 1, BC = x and AC = y. Then cos ∠ ABC = ______________________.
(1, x, y, $\frac{x}{y}$)

2. If $\cos^2 \theta = \frac{1}{2}$ and θ is an acute angle, then the value of θ is ________.
(0°, 30°, 45°, 90°)

3. $\cos (90° - \theta) \sin \theta + \sin (90° - \theta) \cos \theta =$ ________________.
(2, $2 \sin \theta \cos \theta$, $2 \cos 90° \sin 90°$, 1)

Objective : UNDERSTANDING

4. If the three sides of a triangle are $\sin \theta$, $\cos \theta$ and 1, then the triangle must be____________ triangle.
(a right-angled, an isosceles, an equilateral)

5. If the two equal sides of an isosceles triangle be $\sin \theta$ and $\cos \theta$, then the value of θ is______________________.
(60°, 30°, 45°)

Objective : APPLICATION

6. If the two adjacent sides of a rectangle be of magnitudes $\sqrt{\cos \theta}$ and $\sqrt{\sin \theta}$, then the square of its diagonal will always be____________.
(equal to $\sqrt{2}$, less than $\sqrt{2}$,greater than $\sqrt{2}$, less than or equal to $\sqrt{2}$)

7. The value of $\cos^2 \theta$ may be represented by________________.
$\left(\frac{4ab}{(a+b)^2} (a \neq b), \frac{(a+b)^2}{4ab} (a \neq b)\right)$

Objective : SKILL

8. A right-angled triangle ABC is formed such that the magnitudes of the hypotenuse AB is tan x and those of AC and BC are respectively 1 and $\sqrt{2}$ tan y (with suitable values of x and y). Another right-angled triangle ADC is formed with AC as the hypotenuse. Then the two other sides of $\triangle$ ADC may be ____________________.

(sin x and $\frac{1}{\sqrt{2}}$ cos y, $\sqrt{2}$ sin x and cos y)

CHAPTER X

UNIT VII : LINES AND ANGLES

FORM : Essay Type

Objective : KNOWLEDGE

1. Prove that the sum of three angles of a triangle is two right angles. Hence, show that if a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Objective : UNDERSTANDING

2. In a $\triangle$ ABC, $\angle B > \angle C$. L is a point on BC such that AL is the internal bisector of $\angle BAC$. AM is drawn perpendicular to BC, M being a point on BC. Prove that $\angle LAM = \frac{1}{2}(\angle B - \angle C)$.

3. Prove that the angle between the internal bisectors of two adjacent angles of a quadrilateral is equal to half of the sum of two remaining angles of the quadrilateral.

Objective : APPLICATION

4. In a triangle ABC, the base angles B and C are bisected internally by BO and CO respectively. Show that $\angle BOC = 90° + \frac{\angle A}{2}$. Prove also that if BO and CO are the external bisectors of the base angles, then $\angle BOC = 90° - \frac{\angle A}{2}$.

5. Prove that a triangle must have at least two acute angles.

Objective : SKILL

6. Prove that if two lines are perpendicular to two other lines, each to each, the acute angle between the first pair is equal to the acute angle between the second pair.

7. The side BC of the triangle ABC is produced to L. The internal bisectors of ∠ABC and ∠ACL meet at T. Prove that $\angle BTC = \frac{1}{2} \angle BAC$.

8. The lines OA, OB, OC and OD are such that $OA \perp OB$, $OC \perp OD$ and $\angle AOC = \theta$ ($<90°$). P is a point within ∠BOC, but not on OC or OB. M and N are two points on OC and OA respectively such that $PM \perp OC$ and $PN \perp OA$. Prove that $OM^2 + PM^2 = ON^2 + PN^2$.

FORM : Short Answer Type

Objective : KNOWLEDGE

1. With reference to fig. 9, write down the value of the angle a.

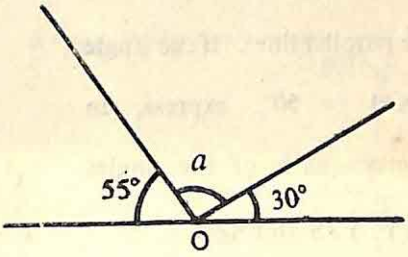


Fig. 9.

2. Let AB and CD intersect each other at 0. If ∠AOD = 80°, find the remaining three angles.

3. AB and CD intersect each other at 0. Of the four angles AOD, BOD, BOC and AOC, which are vertically opposite? State the relations between such angles.

4. With reference to fig. 10, find all the pairs of alternate and corresponding angles.

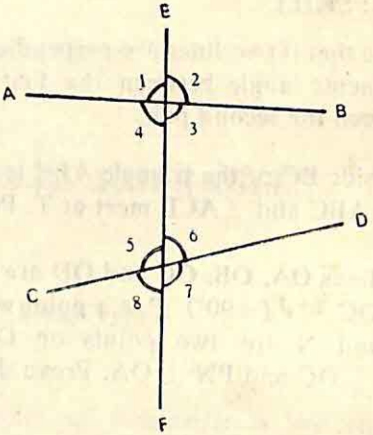


Fig. 10.

5. In fig. 11, LM, PQ and RS are parallel lines. If the angle AXM = 50°, express, in degrees, each of the angles XYP, YZS and SZB.

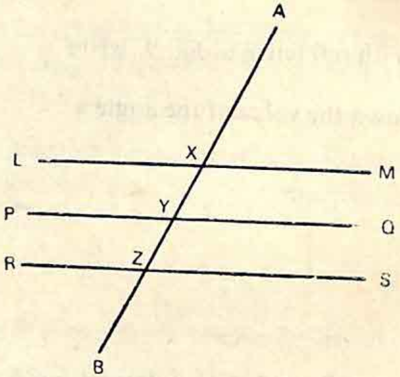


Fig. 11.

6. The opposite sides of a quadrilateral are parallel; if one angle of the quadrilateral is 60°, find the other angles.

7. Find an interior angle of a regular hexagon.

Objective : UNDERSTANDING

8. Let AB and CD be lines which intersect at O. If the sum of the angles AOC, COB and BOD is 270°, find each of the four angles at O.

9. If a line meets two or more parallel lines and is perpendicular to one of them, state with reasons, how it is related to other lines.

10. The sum of the angles at the base of a triangle is 162° and their difference is 60°. Find all the angles of the triangle. What type of triangle is it ?

11. Let ABC be a triangle and D is a point inside it. Prove that $\angle BDC > \angle BAC$.

12. If each of the angles of a regular polygon is 156°, find the number of sides of the polygan.

Objective : APPLICATION

13. In fig. 12, which lines are parallel and why ?

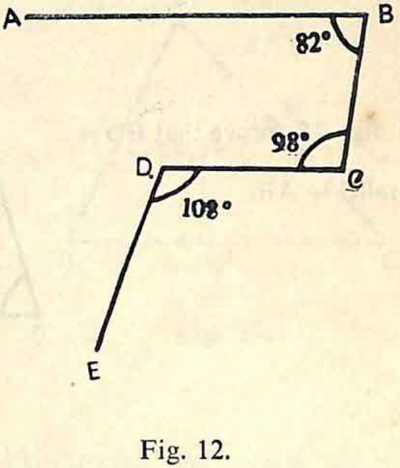


Fig. 12.

14. In fig 13, which lines arc parallel and why ?

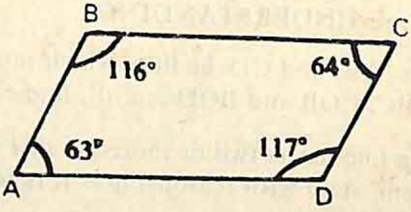


Fig. 13.

15. In fig. 14, if AB is parallel to DE, prove that

$\angle ABC + \angle CDE = 180° + \angle BCD$

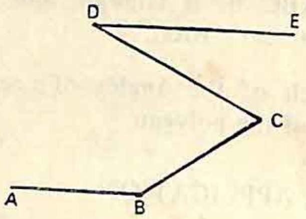


Fig. 14.

16. In fig. 15, prove that ED is parallel to AB.

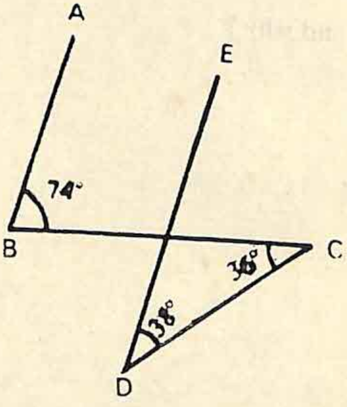


Fig. 15.

17. To find the sum of the angles of the triangle ABC, BC is produced to D and CE is drawn parallel to AB (see fig. 16). Give justification of the above construction. If, on the other hand, AB were produced to F, what parallel would you draw?

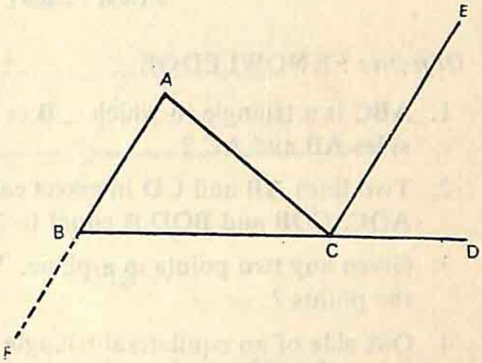


Fig. 16.

18. Give reasons to show that it is not possible to draw two distinct perpendiculars to a line from a given point outside the line.

19. A triangle ABC is drawn on a piece of paper (see fig. 17). It is cut into three parts along the dotted lines shown in the figure. Suggest how you can arrange the three parts to demonstrate that the sum of angles of a triangle is equal to two right angles.

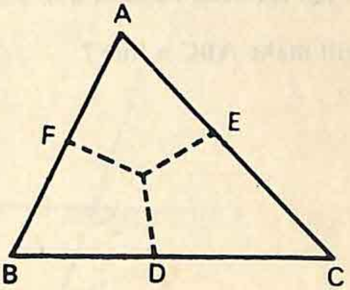


Fig. 17.

Objective : SKILL

20. The base of a triangle is produced bothways. Find by how much is the sum of the exterior angles, thus formed, greater than two right angles if the angle opposite to the base of the triangle is 30°.

FORM : Short Answer Type

Objective : KNOWLEDGE

1. ABC is a triangle in which $\angle B = \angle C$. What is the relation between the sides AB and AC ?
2. Two lines AB and CD intersect each other at O. If the sum of the angles AOC, COB and BOD is equal to 274°, what is the value of $\angle AOC$?
3. Given any two points in a plane. What is the number of lines containing the points ?
4. One side of an equilateral triangle is extended. What is the value of the exterior angle ?
5. Write down the sum of the interior angles of a regular pentagon. will the result be different if the pentagon is not regular ?
6. In fig. 18, what value of $x + y$ will make ABC a line ?

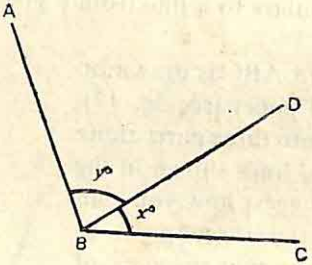


Fig. 18.

7. Let l be a line and P a point on l. How many lines through P and parallel to l can be drawn on the plane of l and P ?
8. Can a triangle have
   (i) all angles less then 60° ?
   (ii) all angles greater than 60° ?
   (iii) all acute angles ?
   (iv) two obtuse angles ?

9. In fig 19, AB ∥ CD. What is the value of ∠GHC ?

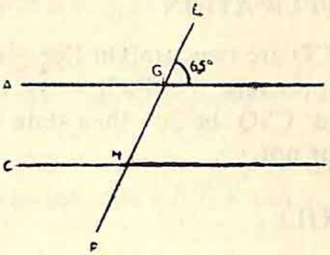


Fig. 19.

Objective : UNDERSTANDING

10. How many triangles can be drawn having its angles as 43°, 64° and 73° ?
11. A number of rays are originating from a point O. What is the sum of all the consecutive angles thus formed ? If the number of rays are increased further, will the sum of the angles increase ?
12. In a triangle ABC, a line LM is drawn through A parallel to BC. What is the difference between the angles CAM and BAL in terms of angles B and C ?
13. With reference to fig. 20, write down the value of x so that the lines AB and CD may be parallel.

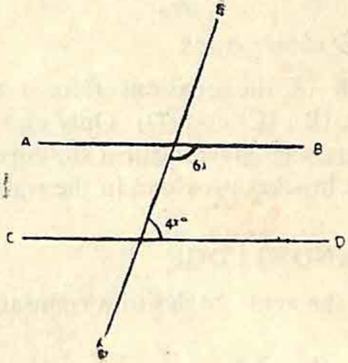


Fig. 20.

14. A regular polygon of x sides has each interior angle equal to y°. Express this statement symbolically in the form of an equation.

Objective : APPLICATION

15. AB and CD are two straight lines and the line PQ intersects them at R and S respectively. If $\angle PRB = 80°$ and the difference between the angles DSQ and CSQ be 20°, then state whether the lines AB and CD will be parallel or not.

Objective : SKILL

16. Construct two angles such that the sides of one angle are perpendicular to the sides of the other, each to each, and such that the two angles are equal.

17. $\angle ACB$ is a right angle. Join AB and take a point D on AB such that $CD \perp AB$. Out of all the angles, thus formed, write all the pairs of angles (without having a common side) each pair of which is such that the sides of one angle of the pair are respectively perpendicular to the sides of the other angle.

FORM : **Objective Type**

(i) *Multiple choice type* :

In each of the quations (No. 1 to 6), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right hand margin.

Objective : KNOWLEDGE

1. One of the acute angles of a right-angled triangle is 58°. The other acute angle is
   (A) less than 32°
   (B) greater than 32°, but less than 90°
   (C) 32°
   (D) none of the above
   ( )

2. The sum of all the internal angles of a regular hexagon each of whose side is 8 cm. is
   (A) 120° (B) 720°
   (C) 960° (D) none of the above ( )

Objective : UNDERSTANDING

3. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is
   (A) a right-angled triangle
   (B) an isosceles triangle
   (C) an equilateral triangle
   (D) an obtuse-angled triangle ( )

4. In fig. 21, the value of x is
   (A) 70°
   (B) 130°
   (C) 80°
   (D) 110° ( )

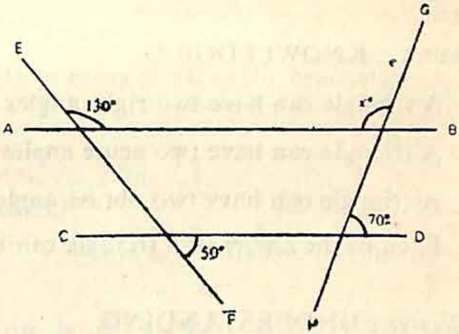


Fig. 21.

Objective : APPLICATON

5. In fig. 22, if $\angle DAB > 45°$, then $\angle DCE$ is equal to
   (A) $\angle DAB$
   (B) $\angle DEC$
   (C) $180° - \angle DAB$
   (D) none of the above

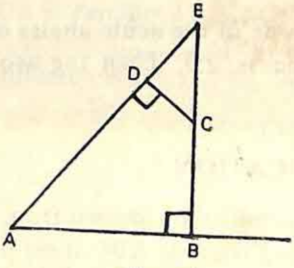


Fig. 22.

Objective : SKILL

6. One of the angles of a triangle is 136°. Then the angle between the bisectors of the acute angles of the triangle is
   (A) 116°
   (B) 44°
   (C) 158°
   (D) none of the above ( )

(ii) *True-false type* :

In the following statements marked with 1, 2, 3, etc., write "T" against each true statement and "F" against each false statement.

Objective : KNOWLEDGE

1. A triangle can have two right angles.
2. A triangle can have two acute angles.
3. A triangle can have two obtuse angles.
4. Each of the angles of a triangle can be less than 60°.

Objective : UNDERSTANDING

5. ABC is a triangle and M and N are two points on AB and AC respectively such that $\angle AMN = \angle ACB$. Then $\angle ANM = \angle ABC$.
6. The ratio of one of the acute angles of a right-angled triangle and its exterior angle is 2:3. Then the two acute angles of the triangle are 72° and 18°

Objective : APPLICATION

7. Let the perpendiculars drawn from the vertices to the opposite sides of an acute-angled triangle ABC meet at O. Let L,M,N be the feet of the perpendiculars on BC, CA and AB respectively. Then $OM = CM \tan A$.

Objective : SKILL

8. OX and OY are two perpendicular lines intersecting at O. These two lines are then rotated through an angle θ about the point O in the anti-clockwise direction. Let these two new lines be Ox and Oy respectively. Take a point P such that it may be above both the lines Ox and OX. PN and PM are drawn perpendiculars to Ox and OX respectively where N and M are points on Ox and OX respectively. Then
   $OM = ON \cos\theta - PN \sin\theta$.

9. Two angles $\angle AOB$ and $\angle COD$ are said to be qually inclined to each other if $\angle AOC = \angle BOD$. If two angles are equally inclined to each other, then they will always have a common bisector.

(iii) *Sentence-completion type* :

Fill up the blanks with appropriate answers given in the brackets :

Objective : KNOWLEDGE

1. There are________________points on a line.
   (only two, only three, more than three)

2. On a blackboard,________________line segments can be drawn.
   (more than four, only four)

3. If the sum of two angles of which one is acute is 180°, then each of them is said to be____________________
   (supplementary, complementary, right-angled)

4. The side AO of an acute angle AOB is produced to C; OY is the bisector of the angle BOC; then OY is ______________of the angle AOB.
   (an internal bisector, an external bisector, a side)

5. Two distinct lines in a plane are either parallel or they intersect in ________________point.
   (a single, more than one)

6. XX′ and LM are two parallel lines;pe rpendiculars are drawn to X′X from three points A, B, C on LM. Then the perpendiculars are ______.
   (parallel, not parallel)

Objective : UNDERSTANDING

7. If the angles of a triangle are in the ratio 1: 1: 2, then the triangle is ______________________.
   (isosceles, right isosceles, equilateral).

8. Angles ABC and DBE are such that they have the same bisector BO and $\angle DBE < \angle ABC$. Then
   $\angle ABE$ ______________ $\angle DBC$.
   $(<, =, >)$

9. Take an acute angle $\angle AOB$. $CO \perp AO$ and $DO \perp BO$. Then $\angle AOB$ ___________ $\angle COD$.
   $(<, >, =)$

10. The internal and external bisectors of an angle form______________.
    (a right angle, an acute angle, an obtuse angle)

Objective : APPLICATION

11. A straight line AOB of which O is the middle point is drawn on a plane paper, which is then folded about O, so that OA falls on OB. Then the crease left in the paper and OB make ________________________.
    (a right angle, an acute angle, an obtuse angle)

Objective : SKILL

12. There are six persons standing at six points O, A, B, C, D, E such that no three points are collinear. Each person wishes to go to the next consecutive point on his right in the shortest path i.e., for example, the person at O wishes to go to A along OA, that at A to B along AB and so on. Assume that each person walks with the same constant speed μ metre/sec. They started at the same time and reached their destinations at the same time after t secs. Then the angle between the direction of one person and the next person on his right will be ______________.
    (120°, 60°, 30°)

CHAPTER XI

UNIT VIII : CONGRUENCE

FORM : **Essay Type**

Objective : KNOWLEDGE

1. Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that O is the point of bisection of both the lines AB and CD.
2. In a triangle ABC, D and E are the middle points of AB and AC respectively. If DE = x cm., prove that BC = 2x cm. and $\angle$ ADE = $\angle$ ABC.

Objective : UNDERSTANDING

3. A,B,C are three non-collinear points in a plane. Find a point in that plane which is equidistant from A,B,C. Justify your answer with arguments.

4. ABC is an isosceles triangle with AB and AC as its equal sides. AD is drawn perpendicular to the base BC (see fig. 23). To prove that AD bisects BC, a student proceeded as follows :

In triangles ABD and ACD,

AB = AC (given),

$\angle$ ABC= $\angle$ ACB ($\because$ AB=AC)

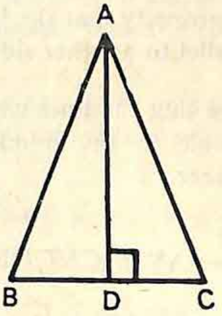


Fig. 23.

and AD is common.

$\therefore \triangle ABD \cong \triangle ACD$

$\therefore BD = CD$

$\therefore$ AD bisects BC.

What is the defect in the above arguments ? Give a correct proof.

5. P,Q,R,S are mid-points of the sides AB, BC, CD and DA respectively of the quadrilateral ABCD. Prove that PQRS is a parallelogram. Prove further that PQRS is a rhombus if AC = BD.

6. P is a point on the bisector of the angle ABC. If the line through P parallel to AB meets BC at Q, prove that the triangle BPQ is an isosceles triangle.

7. PQ and RS are two equal and parallel line segments. Any point M not on PQ or RS is joined to Q and S, and lines through P parallel to QM and through R parallel to SM meet at N. Prove that MN is equal and parallel to PQ.

8. Let l,m,n be three parallel lines. Let p be a transversal intersecting the lines l,m and n at A,B and C respectively such that AB = BC. Let q be another transversal intersecting the lines l, m and n at D, E and F respectively. Join DB and produce it to meet the line n at G. Prove that DB = BG. Hence prove that DE = EF. (You can assume the property that the line through the mid-point of one side of a triangle parallel to another side bisects the third side).

9. Show that the lines which join the extremities of the base of an isosceles triangle to the middle points of the opposite sides are equal to one another.

Objective : APPLICATION

10. In fig. 24, ABCD is a parallelogram and triangles BXC and DYA are such that BX = DY and CX = AY.

Prove that :

(i) BX ∥ DY.

(ii) XY and BD bisect each other at O.

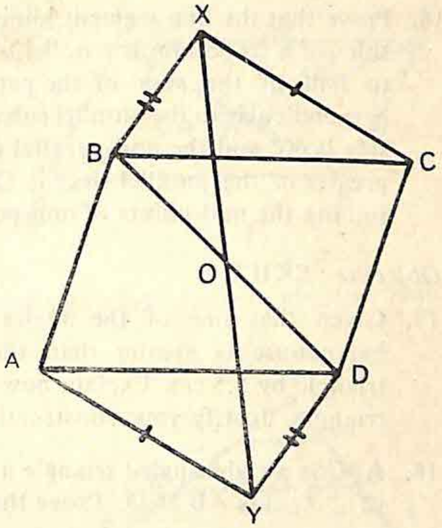


Fig. 24.

11. PQRS is a trapezium, PQ, SR being the parallel sides, and PS, QR the equal sides. The angles at R and S are acute. Prove that ∠ R = ∠ S the diagonals are equal.

12. A,B,C,D are four points in order on a line, and AB = BC = CD. A rhombus BCEF is drawn on BC as base. Prove that DE is parallel to CF. Hence deduce that DE and AF, if produced, would meet at right angles.

13. Let AB and CD be respectively the smallest and the largest sides of the quadrilateral ABCD. Between ∠ C and ∠ A, which is greater ? Justify your answer with arguments.

14. ABC is a right-angled triangle where ∠ ABC = 90°. D is the middle point of AC. Prove that BD = $\frac{1}{2}$ AC.

15. A trapezium ABCD is said to be isosceles if the non-parallel sides AD and BC are equal. Prove that the mid-points of the sides of an isosceles trapezium are the vertices of a rhombus.

16. Prove that the line segment joining the middle points of the non-parallel sides of a trapezium is parallel to either of the parallel sides and is equal to half of the sum of the parallel sides. A diagonal of a trapezium is perpendicular to the parallel sides. An acute angle on the smaller parallel side is 60° and the non-parallel side adjoining the angle is 7 cm. If the greater of the parallel sides is 12 cm., find the length of the line segment joining the mid-points of non-parallel sides of the trapezium.

Objective : SKILL

17. Given that one of the angles of a right-angled triangle is 60° and the hypotenuse is greater than the smaller of the other two sides of the triangle by 5.5 cm. Explain how you can construct any such right-angled triangle. Justify your constructions with arguments.

18. ABC is a right-angled triangle and $AB = AC$. CD, the internal bisector of $\angle$ C cuts AB at D. Prove that $AC + AD = BC$.

FORM : **Short Answer Type**

Objective : KNOWLEDGE

1. In triangles ABC and DEF, $AB = DE$ and $BC = EF$. What other condition(s) do you find necessary to prove that the triangles are congruent ? State the conditions under which two triangles are necessarily congruent.

2. P is any point on the internal bisector of the angle BOA. E and F are two points on OA and OB respectively such that $PE \perp OA$ and $PF \perp OB$. Prove that triangles OPE and OPF are congruent.

3. ABC is an isosceles triangle with $AB = AC$. If D is the mid-point of BC, show that triangles ABD and ADC are congruent.

4. Find the error, if any, in the following statements : Let ABCD be a parallelogram in which the diagonals AC and BD intersect at O (see fig. 25).

In the triangles AOB and COD,
$\angle OAB = \angle OCD$,
$\angle AOB = \angle COD$.
and $AB = CD$.
$\therefore \triangle AOB \cong \triangle COD$.
Therefore, $OA = OD$.
But $OA = \frac{1}{2} AC$
and $OD = \frac{1}{2} DB$
$\therefore AC = DB$.

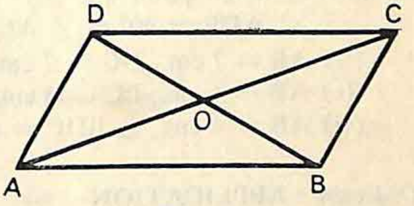


Fig. 25.

In other words, the two diagonals of a parallelogram are always equal.

5. In a triangle ABC, LM, NR, ST are drawn parallel to the base BC such that $AL = LN = NS = SB$. (see fig. 26). show that $LM = \frac{1}{4} BC$.

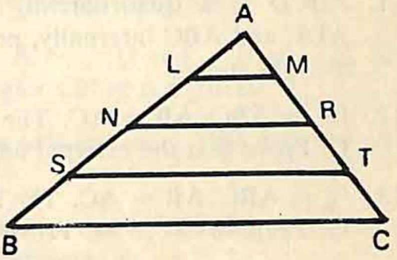


Fig. 26.

6. In $\triangle PQR$, $PQ = QR$. L,M,N are respectively the middle points of the sides PQ, QR and RP. Prove that $LN = MN$ and $\angle PLM = \angle PNM$.

7. If the internal bisector of the angle at the vertex of a triangle bisects the base, then prove that the triangle is an isosceles.

8. Let ABC and KDY be two triangles such that $\angle A = \angle D$, $\angle C = \angle Y$ and $BC = DY$. Are the triangles ABC and KDY always congruent ? Explain your answer.

9. ABCD is a quadrilateral. State, with reasons, in which of the following cases, the quadrilateral ABCD cannot be a parallelogram :
(i) $AB = 8$ cm., $DC = 8$ cm., $\angle ADC = 53^\circ$ and $\angle BCD = 125^\circ$.
(ii) $AB = 5{,}9$ cm., $DC = 5.9$ cm., $\angle ADC = 62^\circ = \angle ABC$.

(iii) E is a point on AB such that AE = 6 cm., EB = 2 cm., $\angle$ ADE = 40° = $\angle$ AED, BC = 7 cm.

(iv) AB = 7 cm., DC = 7 cm., $\angle$ DAB = 130°, $\angle$ ADC = 50°.

(v) AB = 6 cm., BC = 4 cm., CD = 6 cm. and DA = 3.9 cm.

(vi) AB = 6 cm., $\angle$ BDC = 40°, $\angle$ DBC = 36°, AD $\perp$ AB.

Objective : APPLICATION

10. ABCD is a parallelogram and X, Y are the mid-points of the opposite sides AD and BC respectively. Indicate how you can show that BX and DY trisect the diagonal AC.

11. ABCD is a quadrilateral. If the diagonal DB bisects both the angles ADC and ABC internally, prove that DB and AC will bisect each other at right angles.

12. In $\triangle$ ABC, AB = AC. The internal bisectors of $\angle$ B and $\angle$ C meet at O. Prove that the external angle of $\angle$ ABC is equal to $\angle$ BOC.

13. In $\triangle$ ABC, AB = AC. The internal bisectors of $\angle$ B and $\angle$ C meet at O. Produce BO to M. Prove that $\angle$ MOC = $\angle$ ABC.

Objective : SKILL

14. Suppose AB is a line segment and CD is any line. If M and N are two points on CD such that AM $\perp$ CD and BN $\perp$ CD, then the length MN is called the projection of AB on CD. In particular, if AB intersects CD at A, then the length AN will be called the projection of AB on CD. Take two line segments OA and OB which are perpendicular to each other. Denote by the line segment MN, the sum of the projections of OA and OB on CD which makes an angle 45° with OA and not the bisector of $\angle$ AOB. Find the projection of MN on the line which coincides with OB, in terms of the lengths OA and OB.

15. You are given a ruler with parallel sides, but with broken ends. State briefly how you will draw a perpendicular on a line at a given point on it with the help of this ruler and a pencil only. (You can use the property that the two diagonals of a rhombus are perpendicular to each other).

16. State briefly how you will draw a triangle if the middle points of its sides are given.

FORM : **Very Short Answer Type**

Objective : KNOWLEDGE

1. In the two triangles ABC and DEF, AB = DE, BC = EF. Name two angles from two triangles that must be equal so that the two triangles are congruent.

2. In two triangles ABC and DEF, ∠ A = ∠ D, ∠ B = ∠ E and ∠ C = ∠ F. Will the two triangles be always congruent ?

3. In two triangles ABC and DEF, ∠ A = ∠ D and ∠ B = ∠ E. State another relation so that the two triangles will be congruent.

4. ABCD is a parallelogram. If the diagonals are equal, then give the value of the ∠ ABC.

5. ABCD is a parallelogram. If its diagonals are perpendicular to each other, then state the specific name of the parallelogram.

6. ABCD is a parallelogram. AC and BD are diagonals intersecting at O. OA = 3 cm. and OD = 2 cm. State the length of AC and BD.

7. In the triangle ABC, D is the mid-point of AB and DE drawn parallel to BC cuts AC at E. State the value of the ratio AC : CE.

8. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, the triangles must be congruent". Is the statement true ?

9. If two angles of one triangle are equal to two angles of another triangle, then
   (i) will the pair of third angles be equal ?
   (ii) will the sides of one triangle be always equal to the corresponding sides of the other triangle ?

10. In the triangles ABC and DEF,
$AB = x$, $AC = y$ and $\angle BAC = 60°$;
$DE = x$, $DF = y$ and $\angle EDF = 60°$.
Write the ratio BC : EF.

11. A square is a quadrilateral in which the diagonals are equal and bisect each other at right angles. State the name of another quadrilateral in which the diagonals bisect each other at right angles, but they are unequal.

Objective : UNDERSTANDING

12. ABCD is a quadrilateral such that $AB = CD$. State another relation between AB and CD such that ABCD is a parallelogram.

13. In fig. 27, D is the mid point of AB. If $AE = x + 7$ and $CE = 3x + 3$, write down the value of x so that DE is parallel to BC.

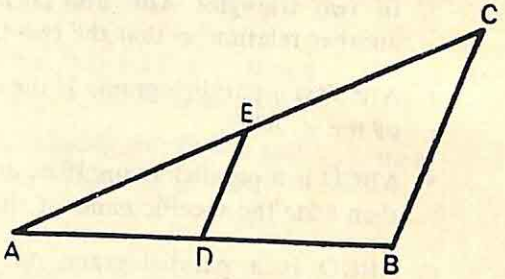


Fig. 27.

14. A right-angled triangle has two equal sides. Write down the ratio of its angles.

15. In the triangle ABC, AC and BC are produced to D and E respectively such that $AC = DC$ and $BC = CE$. Will AB intersect ED when produced in both the directions ?

16. Can all the interior angles of a quadrilateral be obtuse ? If not, why ?

Objective : APPLICATION

17. Is it possible to construct a quadrilateral which has three obtuse exterior angles ?

18. Two isosceles triangles ABC and ADC are constructed on the common base AC on opposite sides of AC. The figure ABCD thus formed is called a "deltoid". A deltoid ABCD is constructed such that $\angle BAC = \angle CAD = 45°$. Name two properties of a rhombus which are common with those of the deltoid.

19. Each of the altitudes drawn through the vertex of a rhombus to the opposite side forms an angle of 30°. What are the angles of the rhombus?

Objective : SKILL

20. Is it possible to construct a parallelogram if two diagonals are 6 cm. and 10 cm. respectively and one of the sides is 8 cm. ? If so, what is the length of the other side of the parallelogram ?

21. Is it possible that all the sides of a parallelogram and one of its diagonals may be equal ? If so, what is the angle between the two diagonals ?

22. Is it possible to construct a parallelogram such that one of its diagonals is perpendicular to two sides of the parallelogram and is equal to each of these sides ? If so, what are the angles of the parallogram ?

FORM : **Objective Type**

(i) *Multiple choice type*:

(a) *Only wrong answer type* :
In each of the questions (No. 1 to 8), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right-hand margin.

Objective : KNOWLEDGE

1. In a triangle, if one of the angles is equal to the sum of the other two angles, then the triangle is
   (A) a right-angled triangle
   (B) an obtuse-angled triangle

(C) an acute-angled triangle whose two sides are equal
(D) an acute-angled scalene triangle ( )

Objective : UNDERSTANDING

2. ABC and DBC are two triangles such that ∠ ABC = ∠ ACB, ∠ A = 36°, BD bisects ∠ ABC and CD is parallel to BA. Then ∠ BDC and ∠ BAC are
   - (A) complementary but unequal
   - (B) supplementary but unequal
   - (C) equal
   - (D) none of the above

   ( )

3. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form a
   - (A) square
   - (B) rhombus
   - (C) rectangle
   - (D) any other parallelogram

   ( )

4. The figure obtained by joining the consecutive mid-points of the sides of any rhombus is always
   - (A) a rhombus
   - (B) a rectangle
   - (C) a square
   - (D) none of the above

   ( )

Objective : APPLICATION

5. H and K are the mid-points of the sides AB, AC of △ ABC and O, any point on BC, is joined to A. If L, M are the mid-points of OB and OC respectively, then HKML is always
   - (A) a square
   - (B) a rectangle
   - (C) a rhombus
   - (D) any other parallelogram ( )

6. H and K are the mid-points of the sides AB and AC respectively of △ ABC. HK is produced to L. To prove that CL is equal and parallel to HA, we need an additional condition which is
   (A) ∠ HAK = ∠ KLC
   (B) AK = KL
   (C) HK = KL
   (D) ∠ AHK = ∠ KCL ( )

7. The figure formed by joining the consecutive mid-points of the sides of a quadrilateral ABCD is a square if and only if
   (A) ABCD is a rhombus
   (B) the diagonals of ABCD are equal
   (C) the diagonals of ABCD are equal and perpendicular
   (D) the diagonals of ABCD are perpendicular ( )

Objective : SKILL

8. The diagonals AC and BD of a parallelogram ABCD cut at O. If ∠ DAC = 32°, ∠ AOB = 70°, then ∠ DBC is
   (A) 24°
   (B) 86°
   (C) 38°
   (D) 32° ( )

(ii) *Only wrong answer type* :

In each of the questions (Nos. 1 and 2) possible answers are marked with (A), (B), (C), etc. Only one of these answers is incorrect and others are correct. The letter that has been given against the incorrect answer should be written in bold letter in the bracket provided in the right-hand margin.

Objective : KNOWLEDGE

1. The criteria for the congruence of two triangles in the usual symbols are
   (A) SAS
   (B) ASA

(C) SSA
(D) SSS ( )

Objective : UNDERSTANDING

2. In a parallelogram,
   (A) opposite sides are equal
   (B) the sum of any two interior angles formed with a side of the parallelogram is two right angles
   (C) opposite angles are equal
   (D) opposite angles are always bisected by the diagonals
   (E) the diagonals bisect each other ( )

(iii) *Mixed type*:

In the following questions (No. 1 to 3) some statements are given in (A), (B), (C), etc. Write against each statement "A", if it is true in all cases, "S", if it is true in some cases and false in others and "N", if it is true in no case.

Objective : UNDERSTANDING

1. (A) If two parallel lines are cut by a transversal, then a pair of alternate angles are equal.
   (B) If two lines are cut by a transversal, then the lines bisecting a pair of alternate interior angles are parallel.
   (C) In a plane, if a line is parallel to one of two parallel lines, it is parallel to the other.
   (D) In a plane, two lines are either parallel or they intersect.
   (E) If two lines are intersected by a transversal, then the alternate angles are equal.
   (F) It is possible to draw two distinct parallel lines to a line through a point not on it.

2. (A) In a parallelogram, the opposite angles are supplementary.
   (B) If two sides of a quadrilateral are equal and the other two sides are parallel, then the quadrilateral is a parallelogram.

(C) If the diagonals of a parallelogram are perpendicular to each other, the parallelogram is a square.

(D) A diagonal of a parallelogram bisects the opposite angles through which it passes.

(E) If a diagonal of a parallelogram other than a square divides it into two isosceles triangles, the parallelogram is a rectangle.

(F) The figure obtained by joining the mid-points of the sides of a quadrilateral is a parallelogram.

3. (A) The bisectors of any two consecutive angles of a parallelogram meet at right angles.

(B) The diagonals of a rhombus are always equal.

(C) If a diagonal of a quadrilateral divides it into two congruent triangles, the quadrilateral is a parallelogram.

(D) All the angles of a rhombus are equal.

(E) The lines joining a vertex of a quadrilateral and the mid-points of the opposite sides trisect the diagonal through other two opposite vertices of the quadrilateral.

(ii) *True false type* :

In the following statements marked with 1,2,3, etc., write "T" against each true statement and "F" against each false statement.

Objective : KNOWLEDGE

1. The angles of a triangle may be 70°, 56°, 63°.
2. Each of the internal angles of a regular hexaogn is 120°.
3. A trapezium is divided into two congruent triangles by each diagonal.
4. The perpendicular bisector of the base of an equilateral triangle will not pass through its vertex.
5. The diagonal of a rhombus is always equal to its side.

Objective : UNDERSTANDING

6. All equilateral triangles are congruent.

7. An equilateral triangle is isosceles.

8. It is given a correspondence between two triangles such that two sides and one angle of the first triangle are equal to the corresponding parts of the second triangle. The two triangles are then always congruent.

9. If two supplementary angles are equal, each is a right angle.

10. If two triangles are congruent, then they are equiangular.

11. Two triangles are always congruent if two sides and one angle of one are equal to two sides and an angle of the other.

12. Two equiangular triangles are always congruent.

Objective : APPLICATION

13. A diagonal of a trapezium is perpendicular to one of the non-parallel sides and the angle opposite to this diagonal is equal to 40°. If the smaller of the two parallel sides is equal to other non-parallel side of the trapezium, then the angles of the trapezium are 140°, 80°, 100° and 40°.

14. The figure obtained by joining the mid-points of a pair of opposite sides of any quadrilateral other than a parallelogram with the mid-points of the two diagonals of the quadrilateral can always be divided into two congruent triangles by joining the pair of opposite vertices of the figure.

Objective : SKILL

15. The ratio of the angles of a trapezium can be taken, in order, to be equal to 6 : 3 : 4 : 2.

16. The ratio of the angles of a trapezium can be taken, in order, to be equal to 8 : 7 : 13 : 12.

17. ABC is an equilateral triangle. A rhombus BCDE is drawn on the base BC of the △ ABC on that side of BC on which A does not lie such that

the side of the rhombus is equal to the side of the △ ABC. A square EDGF of side equal to the side of the rhombus is then drawn on the side ED such that EF or FE produced passes through A. Then ∠ AEG is equal to 45° or 135° according as the square lies on the same side of the rhombus or on the opposite sides of it.

(iii) *Matching type* :

Objective : KNOWLEDGE

1. Given that △ ABW ≅ △ MKF and both the triangles are scalene. In column I, elements of △ ABW and in column II those of △ MKF are given. Match the elements of column I with those given in column II so that the two elements are equal, in the manner like (N)—(X), where N is a positive integer and X is an English alphabet.

Column I	*Column II*
(1) AB	(A) ∠ M
(2) ∠ B	(B) ∠ F
(3) BW	(C) M F
(4) ∠ W	(D) KF
(5) ∠ A	(E) M K
(6) AW	(F) ∠ K

Objective : UNDERSTANDING

2. ABCD is a parallelogram and P, Q, R and S are points in AB, BC, CD and DA respectively such that AP = CR, BQ = DS. Match the triangles given in column I, which are congruent with the triangles given in column II in the manner like (N) – (X), where N is a positive integer and X is an English alphabet.

Column I	*Column II*
(1) △ APS	(A) △ RSD
(2) △ CDQ	(B) △ QRC
(3) △ PBQ	(C) △ CDS
(4) △ ABQ	(D) △ ABS

3. ABCD is a quadrilateral and E, F, G, H are the middle points of the sides AB, BC, CD and DA respectively. Join EF, FG, GH and HE. The nature of the quadrilateral EFGH will depend on the nature of quadrilateral ABCD. In column I and column II, different natures of the quadrilaterals ABCD and EFGH are given. Match the elements in column I with those in column II in the manner such as (N)—(X), where N denotes a positive integer and X is an English alphabet.

Column I	*Column II*
(1) ABCD is a square	(A) EFGH is a parallelogram other than (B), (C) and (D)
(2) ABCD is a rectangle	(B) EFGH is a rhombus
(3) ABCD is a rhombus	(C) EFGH is a square
(4) ABCD is any other parallelogram	(D) EFGH is a rectangle

4. ABC is a triangle. D, E, F are the mid-points of the sides AB, BC and CA respectively. Join DE, EF and FD. The nature of △ DEF will depend on the nature of △ ABC. In columns I and II, different natures of △ ABC and △ DEF are given. Match the elements in column I with those in column II in the manner like (N) – (X), where N is a positive integer and X is an English alphabet.

Column I	*Column II*
(1) ABC is an equilateral triangle	(A) DEF is a right isosceles triangle
(2) ABC is an isosceles triangle	(B) DEF is an equilateral triangle
(3) ABC is a right isosceles	(C) DEF is a scalene triangle
(4) ABC is a scalene triangle	(D) DEF is an isosceles triangle

5. Two diagonals of a quadrilateral bisect each other in various ways as given in column I. The names of the corresponding figures are given in

column II. Match the elements of column I with those in column II such that the properties mentioned in column I become true in the case of figures mentioned in column II. Match these in the manner like (N)–(X), where N is a positive integer and X is an English alphabet.

Column I	*Column II*
(1) Two diagonals are unequal and bisect each other not at right angles	(A) a square
(2) Two diagonals are unequal and bisect each other at right angles	(B) a rectangle
(3) Two diagonals are equal and bisect each other not at right angles	(C) a rhombus
(4) Two diagonals are equal and bisect each other at right angles	(D) any other parallelogram

(iv) *Sentence completion type* :

Fill up the blanks with appropriate answers given in the brackets.

Objective : KNOWLEDGE

1. Three angles of a triangle are 30°, 60° and 90°. We can construct __________triangle with these angles.
(one, more than one)

2. Two equilateral triangles with equal bases are________ ________ congruent.
(necessarity, not necessarily)

3. Any exterior angle of a triangle is always____________________ each opposite interior angle.
(greater than, equal to)

4. The two diagonals of a rhombus form____________________.
(an obtuse angle, an acute angle, a right angle)

Objective : UNDERSTANDING

5. The figure obtained by joining the mid-points of the consecutive sides of a rectangle is a________________.
(rectangle, square, rhombus)

6. A diagonal always separates a parallelogram into two________________ triangles.
(equilateral, isosceles, congruent)

7. In a parallelogram, any two adjacent angles are always________________.
(equal, supplementary, complementary)

8. If a parallelogram with unequal adjacent sides has one right angle, then it is a____________________.
(square, rectangle, rhombus)

Objective : APPLICATION

9. ABC is an acute-angled triangle and on the sides AB and AC, equilateral triangles APB and AQC are described outside the given triangle. Then the triangles PAC and____________________ are always congruent.
(BQC, BAQ, PBC)

10. BCD and WXY are two triangles such that DS bisects ∠ BDC meeting BC at S and YT bisects ∠ WYX meeting WX at T. Given that (a) △ BCD = △ WXY (b) BD = WY and (C) ∠ BDS = ∠ WYT. Among these given data, the superfluous data in proving that DS = YT are________________.
[(a) and (c), (a) and (b), (b) and (c)]

11. ABCD is a parallelogram in which BC = 2 AB and E, F are the mid-points of AD, BC respectively. If AF and BE cut at G and CE and DF cut at H, then EGFH is always a____________________.
(square, rhombus, rectangle)

Objective : SKILL

12. The two sides of a triangle are AB = 4 cm. and AC = 2.5 cm. and ∠ ABC = 30°. We can draw ______________ triangle (s) with these data.
(only one, two, three, more than three)

13. If the base, the angle at the vertex and the difference of other two sides of a triangle are given, then ______________ triangle can be drawn
(no, a unique, more than one)

CHAPTER XII

UNIT IX : CIRCLES

FORM : **Essay Type**

Objective : KNOWLEDGE

1. Prove that the angle subtended by an arc of a circle at its centre is double the angle subtended by it at any point on the remaining part of the circle.
2. Prove that the angles in the same segment of a circle are equal.
3. AB and BC are two equal chords of a circle with centre O. M and N are the feet of the perpendiculars from O to AB and BC respectively. Prove that triangles OMB and ONB are congruent.

Objective : UNDERSTANDING

4. In fig. 28, $\angle BDC = 48°$. Giving suitable reasons, find the angles BAC, BOC, OBC, OCD and BEC.

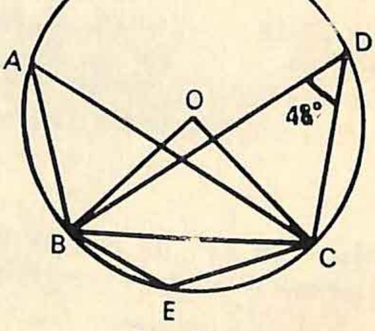


Fig. 28.

5. ABCD is a trapezium of which the oblique sides AD and BC are equal. Two circles are drawn, one through A,B,D and the other through A,D, C. Are the two circles distinct ? Justify your answer with arguments.

6. Two equal chords PQ and RS of a circle intersect at a point A within the circle. Prove that the two chords are equally inclined to the diameter through A.

7. If a straight line intersets two concentrie circles, then prove that the two line segments between the two circles will be equal.

8. Let O be the centre of a circle and AB be a chord. C is a point on the circle such that it lies on that side of AB opposite to O. Join OA, OB and OC and produce BO to a point D on the circle. Prove that
   (i) $\angle$ COD = 2 $\angle$ CBO
   (ii) $\angle$ AOD = 2 $\angle$ ABO
   (iii) $\angle$ AOC = 2 $\angle$ ABC.
   State the theorem that is proved in (iii).

Objective : APPLICATION

9. Any two circles intersect each other at A and B. X, P are points on one circle and Y, Q are those in the second circle and these points are such that the lines XY and PQ pass through A and B respectively. Prove that XP || YQ. Find also the relations between the angles PXA and AYQ in the two cases when (i) PQ and XY intersect within a circle and (ii) PQ and XY do not intersect within any circle.

10. AB is a diameter of a circle and P and Q are two points on the circle such that AP and AQ when produced meet the line through B perpendicular to AB at X and Y respectively. Prove that the quadrilateral PXYQ is cyclic.

11. RSTW is a cyclic quadrilateral. By using the angle sum of triangle RTW and the equality of the angles in the same segment of the circle, prove that a pair of opposite angles of the quadrilateral is supplementary.

12. ABCD is a cyclic quadrilateral. Diagonals AC and BD intersect at P at right angles. E is a point on AD such that EP is perpendicular to AD

and when produced meet BC at F. Prove the following relations.

(i) ∠ EPA = ∠ EDP
(ii) ∠ EPD = ∠ EAP
(iii) ∠ FPB = ∠ FBP
(iv) ∠ FPC = ∠ FCP
(v) BF = FC

13. In a circle with centre O, chord AB is produced to C so that BC equals the radius of the circle. CO is drawn and extended to D. Prove that ∠ AOD = 3 ∠ ACD.

14. A,B,C are three points on a circle. The bisector of ∠ ABC meets the circle again at D. DE is drawn parallel to AB and meets the circle again at E. Prove that BC = DE.

15. ABC is a cyclic triangle. The internal bisector of ∠ A cuts the circumcircle of the triangle at L. CM, the internal bisector of ∠ C, cuts AL at M. Prove that
LB = LC = LM.

16. The cyclic quadrilateral CDEF is such that CF and DE, when produced, meet at B and EF and DC, when produced, meet at A. Prove that the internal bisectors of angles CBD and EAD meet at right angles.

Objective : SKILL

17. Let AOB be a triangle such that angle ABO is obtuse. Semicircles are drawn with OA and OB as diameters such that these semicircles intersect at a point X other than O. Prove that A,B,X are collinear.

18. ABC is a cyclic equilateral triangle. If P be a point on the arc BC opposite to the point A, then prove that AP = BP + CP.

19. Through the points of intersection of two circles, any two straight lines are drawn such that these two lines cut one circle at A and B and the other circle at C and D. Prove that AB ∥ CD.

FORM : **Short Answer Type**

Objective : KNOWLEDGE

1. How many circles can be drawn through one given point ? Give reasons.

2. How many circles can be drawn through two given points ? State reasons for your answer.

3. Is it possible to draw a circle circumscribing a triangle, whatever may be its shape ? State reasons for your answer.

4. "The angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the circumference".
Is the above statement correct ? If not, give the correct statement.

5. ABC is a triangle in which AB = AC. DE is parallel to BC (see fig. 29). To prove that B,C,D, E are concylic, steps for proof are as below :
   (i) $\angle$ EDB + $\angle$ DBC=180°
   (ii) $\angle$ DBC = $\angle$ ECB
   (iii) $\angle$ EDB + $\angle$ ECB=180°.
   Hence, B,C,E,D are concyclic.
   Give reasons for the statements in steps (i), (ii) and (iii).

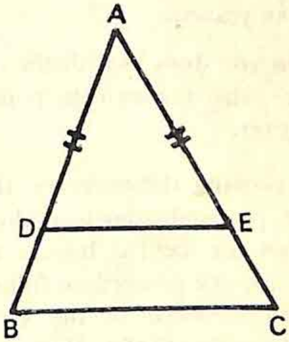


Fig. 29.

Objective : UNDERSTANDING

6. Two congruent circles contain two equal chords. State the relation between the angles which the chords subtend at the respective centres. Justify your answer.

7. In fig. 30, if
$\angle AOB = \angle COD = 60°$,
prove that $AC = BD$.
Detect the superfluous data, if any.

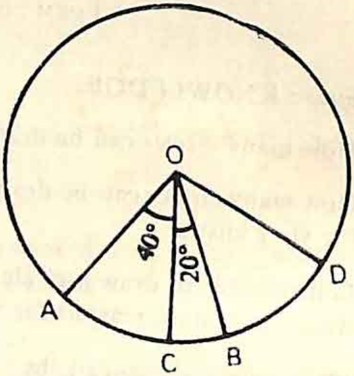


Fig. 30.

8. A,B,C,D are four points on a plane. Is it always possible to locate a point on the same plane, which is equidistant from the given points ? Give reasons.

9. Can you draw two distinct circles—one through the points A,B,C,D and the other through the points A,B,D,E ? Give reasons in support of your answer.

10. In proving the property that the perpendicular to a chord from the centre bisects the chord, one proceeds as follows with reference to the circle with centre O (fig. 31) :
$OA = OB$ (radii),
$\angle OAD = \angle OBD$ ($\because OA = OB$) and OD is common.
$\therefore \triangle OAD = \triangle OBD$.
$\therefore AD = BD$.
Detect the defect, if any, in this proof and make necessary corrections in the proof.

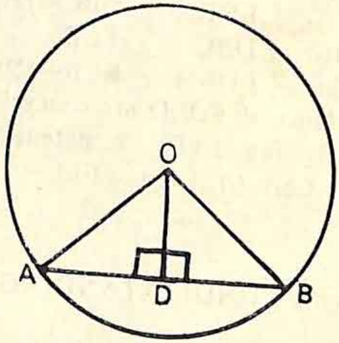


Fig. 31.

11. Going round a circle, two persons A and B traverse paths each of which makes an angle of 60° at the centre of the circle. They move with the same speed. Will they take the same time to describe the paths ? Give reasons.

12. AB is a common chord of two circles with centres P and Q. O is the mid-point of AB. Will it be possible to draw a circle through P,Q and O? Give reasons.

13. In fig. 32, PT bisects the angle QPR. Show that TS bisects angle QSR.

14. State the relation between the opposite angles of a cyclic quadrilateral and hence name the form of a cyclic parallelogram. State the reason in support of your answer.

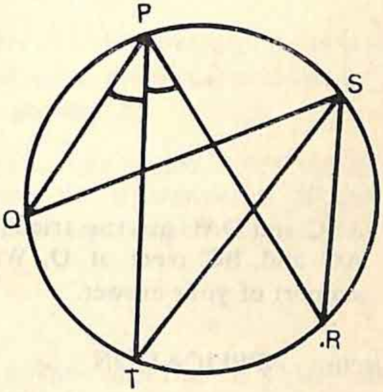


Fig. 32.

15. In fig. 33, give steps to show that the angles of the triangle ABP are equal to those of the triangle CDP.

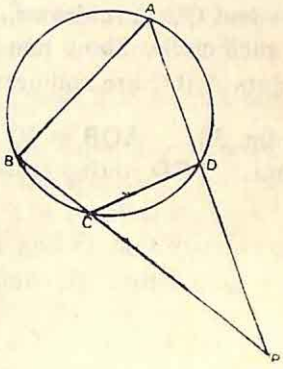


Fig. 33.

16. In fig. 34, AB, AC, AD, DC, DB and CB are chords of a circle. CB and AB are produced to F and G respectively. Find the values of u, v, x, y, z and t as shown in the figure.

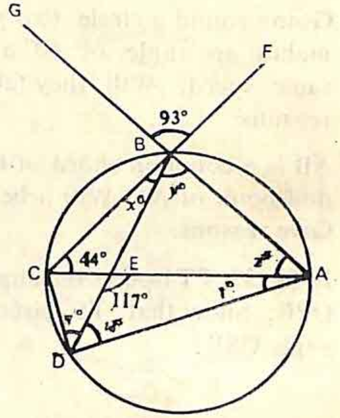


Fig. 34.

17. ABC and DAC are two triangles such that the perpendicular bisectors of AC and BC meet at O. Will OB be equal to OD ? State reasons in support of your answer.

Objective : APPLICATION

18. Two circles intersect at O and B. Through O, two diameters OA and OC are drawn, one in each circle. Show that the points A,B,C are collinear.

19. In fig. 35, ∠ AOB = 70°. Find ∠ BCD stating reasons.

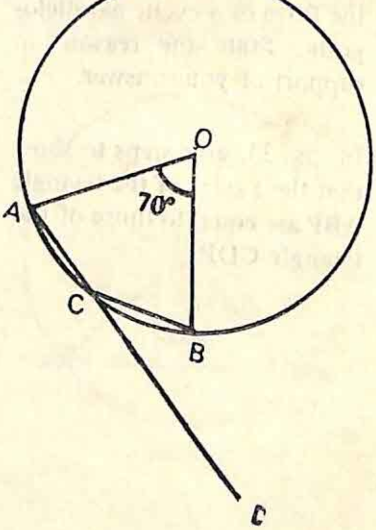


Fig. 35.

20. ABC is a triangle inscribed in a circle. Find, stating reasons, the sum of the angles subtended at the cantre of the circle by the three segments of the circle, cut off by the sides of the triangle.

21. Prove that two chords, not passing through the centre, can not bisect each other.

22. ABCD is a quadrilateral with two sides AB and DC paralled. A circle is drawn with DC as a chord to intersect DA and CB at P and Q respectively. Show that A,B,Q,P are concyclic.

23. Circles are drawn on two adjacent sides of a rhombus with those sides as diameters. Will the common chord, when produced, pass through the opposite angular point ? Justify your answer.

24. ABC is an isosceles right-angled triangle, right-angled at B. O is the mid-point of AC. Prove that the line joining the circumcentres of the two triangles AOB and BOC will bisect OB at right angles.

Objective : SKILL

25. A,B,C,D are four concyclic points in a plane such that D, C are on the same side of the line AB. Five boys X,Y,Z, T and U start from A,C,A, D,A respectively and walk along AC, CB, AD, DB and AB respectively with constant speeds (in m/min) which are respectively in the ratio of $2\sqrt{3} : 2\sqrt{2} : \sqrt{3} : 4 : 10$. The times taken by them to reach the points C,B,D, B and B are $\sqrt{7}$ min, $\sqrt{2}$ min, $\frac{6}{\sqrt{3}}$ min, 2 min and 1 min respectively. If two other boys P and Q start at the same time from C and D respectively along CO and DO where O is the centre of the circle passing through A,B,C, D and if the speed of each of P and Q be equal to that of U, then after how much time will P and Q meet together and what will be the angle between their directions in terms of $\angle$ CAB and $\angle$DBA ?

FORM : Very Short Answer Type

Objective : KNOWLEDGE

1. Three points A,B,C are collinear. Can you draw any circle through A,B, C ? If so, how many circles can be drawn ?
2. ABC is a triangle. The perpendicular bisectors of AB and BC meet at O. Write down the ratio OA : OB : OC.
3. In fig. 36, O is the centre of the circle. If ∠ AOB = 60°, find ∠ ACB.

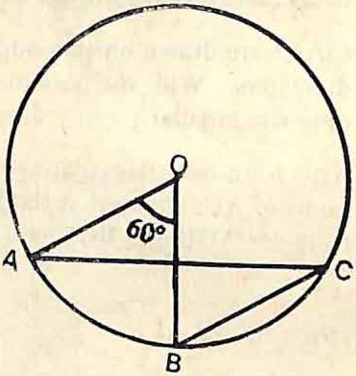


Fig. 36.

4. If ABCD is a cyclic parallelogram, what is the value of angle B ?
5. ABCD is a cyclic quadrilateral. AB is produced to P. If ∠ CBP = 60°, state the value of the angle ADC.
6. ABCD is a quadrilateral. BD is a diagonal and angle ADB is 60°. State the value of ∠ ACB if A,B,C,D are concyclic.
7. Is it possible to find a point P equidistant from three collinear points A, B,C in a plane ?
8. ABC is a triangle. O is a point inside the triangle such that ∠ AOB = x° and ∠ ACB = y°. State the relation between x and y so that O will be the centre of the circle passing through A,B,C.

9. A circle is drawn through three vertices of a rectangle. Will this circle pass through the fourth vertex ?

Objective : UNDERSTANDING

10. AB and CD are two parallel chords of a circle with centre O and M,N are the mid-points of AB and CD respectively. Can you draw any triangle through M,O and N ? If so, how many ?

11. ABC is a triangle such that $\angle C = 90^\circ$. O is the centre of the circle circumscribing the triangle ABC. If OA = 5 cm. what is the length of AB ?

12. ABCD is a quadrilateral such that $\angle A = \angle C = 90^\circ$. Can you draw a circle passing through all the vertices of the quadrilateral ?

Objective : APPLICATION

13. A,B,C,D are four points in a plane such that $\angle ACB = 60^\circ$ and $\angle ADB = 30^\circ$. A circle through A,B,C is drawn. State whether D will be inside, outside or on the circle if D,C lie on the same side of AB.

Objective : SKILL

14. Three pins each of negligible thickness are fixed on a table at points A, C and E which are such that the pairs of points A, C and C, E are the extremities of diameters of two circles such that the common chord makes an angle of 45° with AC. The distance between the pins at A and C and that between the pins at C and E are each equal to a cm. If a boy wishes to surround the pins by a circular thin wire of negligible thickness, then what will be the radius of the circle ?

FORM : **Objective Type**

(i) *Multiple choice type* :

In each of the questions (No. 1 to 10), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right hand margin.

Objective : KNOWLEDGE

1. S is the circumcentre of △ ABC. It is the point of concurrence of
   (A) any bisectors of the sides of the triangle
   (B) the perpendicular bisectors of the sides of the triangle
   (C) the angular bisectors of the angles of the triangle
   (D) the perpendiculars from the vertices to the opposite sides of the triangle

2. In fig. 37, if the chord AB = chord CD, ∠ AOC=40° and ∠ COB=28°, then ∠ BOD is equal to
   (A) 60°
   (B) 40°
   (C) 20°
   (D) 30° ( )

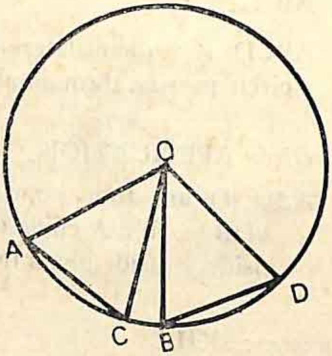


Fig. 37.

3. In fig. 38, the value of x in degrees is equal to
   (A) 36°
   (B) 72°
   (C) 108°
   (D) 180°

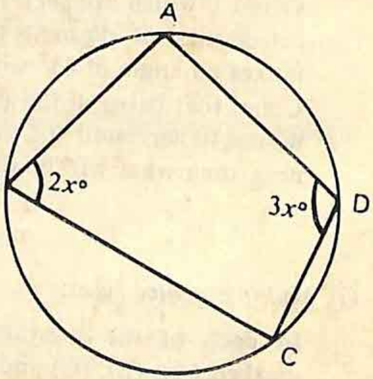


Fig. 38.

4. In fig. 39, the value of the angle C is
   (A) 80°
   (B) 100°
   (C) 20°
   (D) none of the above ( )

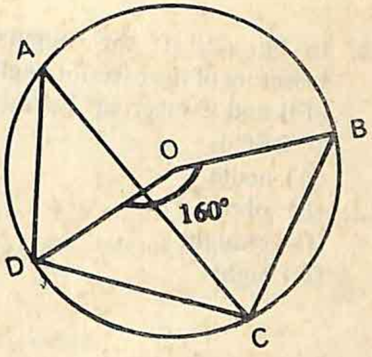


Fig. 39.

5. PQ and RS are equal chords of a circle. The perpendicular bisectors of PQ and RS will
   (A) intersect inside the circle
   (B) intersect outside the circle
   (C) intersect on the circle
   (D) never intersect ( )

6. In fig. 40, BE is the bisector of ∠ CBX. If ∠ ADC=80°, then the value of x is
   (A) 80°
   (B) 40°
   (C) 50°
   (D) 30° ( )

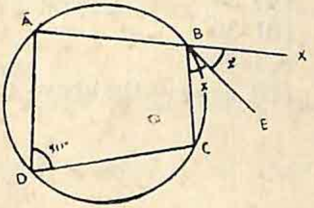


Fig. 40.

7. In fig. 41, O is the centre of the circle. Then ∠ BAC is
   (A) 110°
   (B) 70°
   (C) 105°
   (D) 55° ( )

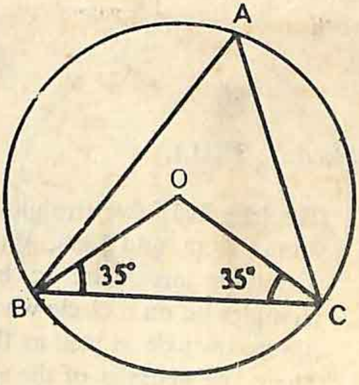


Fig. 41.

8. In fig. 42, if the angular bisectors of the exterior angles of B and C meet at E, then ∠ BEC is
   (A) acute
   (B) obtuse
   (C) straight
   (D) right ( )

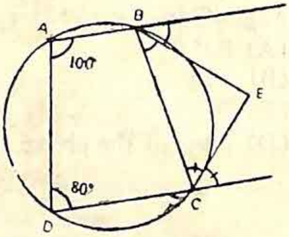


Fig. 42.

9. In fig. 43, ∠ OBA is
   (A) 60°
   (B) 30°
   (C) 90°
   (D) none of the above ( )

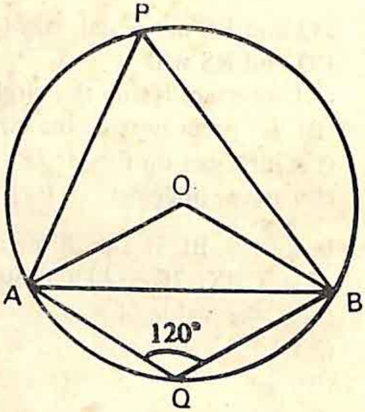


Fig. 43.

Objective : SKILL

10. The lengths of five straight wires of negligible thickness are 4 cm., 4 cm., 3 cm., 3 cm. and 5 cm. With these five wires, two triangular regions of maximum areas are to be surrounded such that all the vertices of two triangles lie on a circle whose diameter is equal to the length of one side of one triangle as well as the length of one side of the other triangle. Then the lengths of the sides of the two triangles and the common side are respectively

(A) 3 cm., 3 cm., 4 cm,; 4 cm., 4 cm., 5 cm.; 4 cm.
(B) 4 cm., 3 cm., 5 cm.; 3 cm., 4 cm., 5 cm.; 5 cm.
(C) 3 cm., 5 cm., 4 cm.; 3 cm., 4 cm., 5 cm.; 3 cm.
(D) 4 cm., 4 cm., 3 cm.; 3 cm., 5 cm., 3 cm.; 3 cm. ( )

(*ii*) *True-false type* :

In the following statments marked with 1,2,3 etc., write "T" against each true statment and "F" against each false statement.

Objective : KNOWLEDGE

1. A circle can pass through three collinear points.
2. An angle in the major segment is obtuse.
3. The perpendicular from the centre of a circle to a chord intersects the chord in the ratio 1 : 1.
4. The sum of a pair of opposite angles of a cyclic quadrilateral is equal to 180°.
5. Any cyclic parallelogram is always a square.
6. If the angle subtended by an arc of a circle is 180° at the centre, then the arc is a semicircle.
7. The perpendicular through the middle point of a chord of a circle will not pass through the centre of the circle.
8. If two chords of a circle are unequal, the longer chord is the one which is nearer to the centre.
9. In fig. 44, chord AB is equal to the chord CD, where O is the centre of the circle.

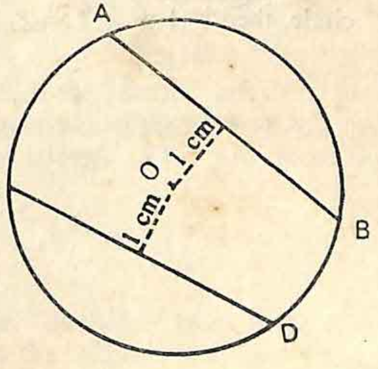


Fig. 44.

Objective : UNDERSTANDING

10. If the line joining the mid-points of two chords (other than diameters) of a circle passes through the centre, then the chords intersect each other.

11. In fig. 45, OA is not equal to the radius of the circle.

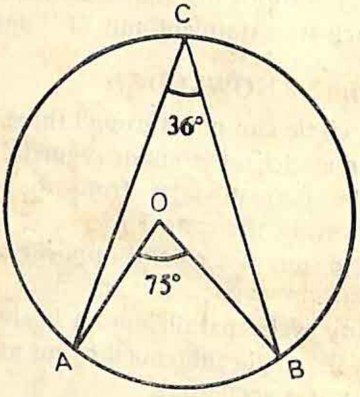


Fig. 45.

12. In fig. 47, if O is the centre of a circle, then $\angle 1 + \angle 2 \neq \angle 3$

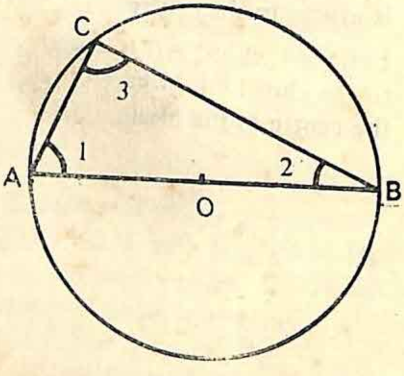


Fig. 46.

13. In fig. 47, BC and PR will intersect each other in the ratio 1 : 1, if $\angle$ BAC = $\angle$ PQR = 90°.

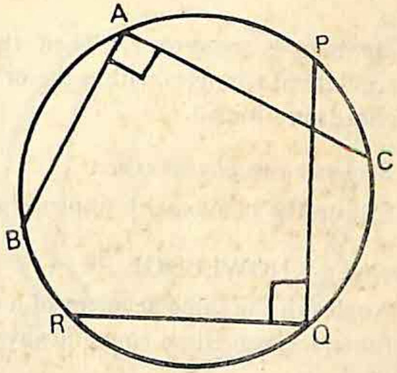


Fig. 47.

Objective : APPLICATION

14. In fig. 48, AB = BC = CD. Therefore, chords AC and BD are equal.

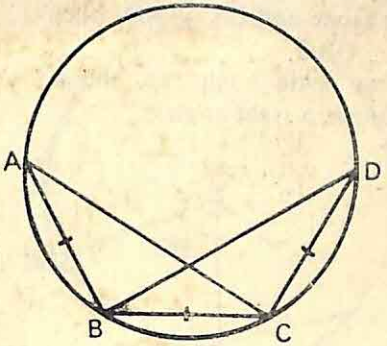


Fig. 48.

15. A triangle ABC is inscribed in a circle. If the internal bisectors of the angles A, B and C intersect the circumference of the circle at X,Y and Z respectively, then the angles of the triangle XYZ are respectively $90° - \frac{A}{2}$, $90° - \frac{B}{2}$ and $90° - \frac{C}{2}$.

Objective : SKILL

16. A circle of radius r is drawn on a plane. Taking any point on the circumference of this circle as centre and r as the radius, six other circles are drawn such that the length of each of the six common chords, thus

obtained, is equal to r. Then the six common chords will form six equilateral triangles within the original circle when the extremities of the chords are joined.

(iii) *Sentence completion type* :

Fill up the blanks with appropriate answers given within the brackets:

Objective : KNOWLEDGE

1. Angles in the same segment of a circle are_______________.
   (unequal, sometime equal, always equal)

2. In fig. 49, if O is the centre of a circle and PR = RQ, then ∠ ORQ = _______________.
   (an acute angle, an obtuse angle, a right angle)

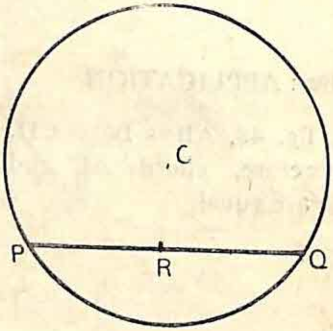


Fig. 49.

3. In fig. 50, ∠ B and ∠ D are _______________
   (complementary, supplementary)

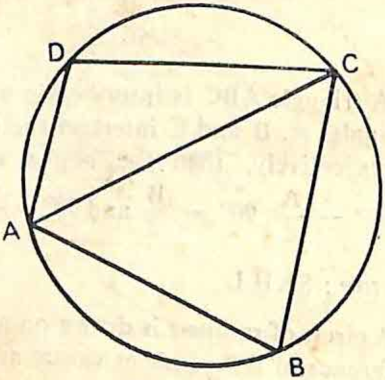


Fig. 50.

Objective : UNDERSTANDING

4. In fig. 51, ∠ PSR = ________________

(greater than 100°, less than 100°, equal to 100°)

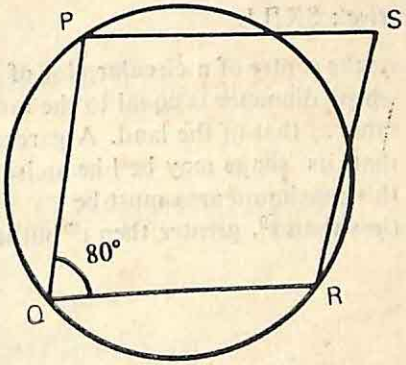


Fig. 51.

5. In fig. 52, O is the centre of a circle and chord CD = chord AB. Therefore ∠ BOD is ________________.
(equal to 70°, less than 70°, greater than 70°)

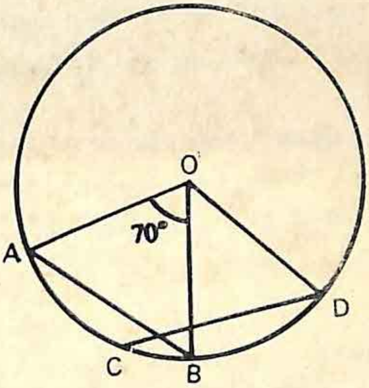


Fig. 52.

Objective : APPLICATION

6. Of all the triangles that can be drawn within a given circle, a triangle with one maximum side must be ________________.
(an equilateral triangle, an isosceles triangle, a right-angled triangle)

7. If the two internal bisectors of two opposite angles of a cyclic quadrilateral intersect the circumference at Q and R, then QR will be ________ ________________.

(a diameter, a chord other than a diameter)

Objective : SKILL

8. At the centre of a circular plot of land of radius r, there is a circular well whose diameter is equal to the radius of the land and whose centre is the same as that of the land. A garden is to be prepared on the land such that its shape may be like an isosceles triangle of maximum area. Then this maximum area must be
   (less than r^2, greater then r^2 but less than $2r^2$, greater than $2r^2$)

CHAPTER XIII

UNIT X : LOCI AND CONSTRUCTIONS

FORM : **Essay Type**

Objective : KNOWLEDGE

1. Prove that the perpendicular bisector of the segment joining two fixed points is the locus of a point equidistant from the two fixed points.
2. Draw an isosceles triangle ABC, given AB = 6 cm., AC = 6 cm. and BC = 8 cm. Draw the medians.
3. AB and CD are two lines which intersect at O such that ∠ AOD = 60°. Draw the angular bisectors of the angles AOD, COA, COB and BOD.
4. Construct a triangle ABC given, BC = 6.5 cm., ∠ B = 60° and AB + AC = 10 cm.

Objective : UNDERSTANDING

5. A,B,C are three non-collinear points in a plane. State the constructions to find another point P on that plane, which is equidistant from A,B,C. Justify your constructions.
6. Prove that the sum of any two sides of a triangle is greater than twice the median which bisects the third side.
7. Prove that the difference of any two sides of a triangle is less than the third side.
8. Construct a triangle ABC whose sides are 4 cm., 6 cm. and 7 cm. Draw its incircle also. Justify your construction.

9. Draw an equilateral triangle of side 6 cm. Circumscribe a circle about it. Justify your construction.

10. Draw a triangle whose sides are 3 cm., 4 cm. and 6 cm. Find its orthocentre. Justify your construction.

Objective : APPLICATION

11. The medians BE and CF of a triangle ABC intersect at O. P and Q are mid-points of BO and CO. Prove that FPQE is a parallelogram.

12. The orthocentre of the triangle ABC is given. Prove that it is also the circumcentre of the triangle PQR obtained by drawing three lines through the three vertices, each parallel to the opposite side of the triangle ABC.

13. Prove that in an equilateral triangle, the circumcentre, the in-centre, the centroid and the orthocentre always coincide.

14. Prove that in an isosceles triangle, the circumcentre, the in-centre, the centriod and the orthocentre lie in one line.

15. Prove that in a right-angled triangle, the circumcentre, the orthocentre and the centroid lie in one line.

16. Prove that the sum of three medians of any triangle is less than its perimeter.

17. Prove that the sum of the distances of three vertices of a triangle from any point within the triangle is greater than half the perimeter of the triangle.

18. The lengths of two diagonals of any quadrilateral are M and N and the length of its perimeter is P. Prove that $\frac{1}{2}P < M + N < P$.

19. Using the result that the perpendiculars drawn from the vertices of a triangle to the opposite sides are concurrent, prove that the perpendicular bisectors of the sides of the triangle are concurrent.

20. Construct a triangle ABC with side BC = 6 cm., AB = 3 cm. and the altitude AD = 2.6 cm. How many such triangles are possible ?

21. Prove that the sum of any two medians of a triangle is greater than the third median.

22. Prove that four times the sum of three medians of a triangle will be greater than three times the perimeter of the triangle.

23. Prove that the distance of a side of a triangle from its circumcentre is half the distance between the orthocentre and the vertex opposite to the side of the triangle.

Objective : SKILL

24. The circumcentre of the triangle PQR is given. Prove that it is also the orthocentre of the triangle ABC obtained by joining the mid-points of the sides of the triangle PQR.

25. Given the base BC, the median AD and the altitude of a triangle ABC; construct the triangle ABC. How many such triangles are possible ?

26. A and B are two points in a plane. State the procedure to find (without a ruler) another point C on that plane so that A,B,C are collinear. Justify your construction.

27. Given the base, the difference of the base angles and the difference between the other two sides of a triangle. Explain how you will construct the triangle by showing and justifying your construction.

FORM : **Short Answer Type**

Objective : KNOWLEDGE

1. Name the fugures that will represent the locus of a point in the following cases :
   (i) The point is always at a distance r from one given point.
   (ii) The point is equidistant from two given points.

2. The bisectors of angles A and B of a triangle ABC are given. Suggest a method to draw the bisector of the angle C without using compass and protractor. Justify your answer.

3. The medians AD and BE of a triangle ABC intersect at O. F is the mid-point of AB. Join FO and CO. State, with reasons, the value of ∠ FOC.

4. The bisectors of angles PQR and QRP of the triangle PQR intersect at I. State the relation between PI and ∠ QPR. Give reasons.

Objective : UNDERSTANDING

5. Write the steps for dividing a line segment in the ratio 1 : 3.

6. Write steps for dividing a right angle into three equal parts without the help of a protractor.

7. Given a right angle. State the steps you will perform to draw an angle of $22\frac{1}{2}°$ with the help of a compass.

8. In the triangle ABC, perpendiculars AD and BE intersect at 0. CO is joined and produced to F to meet AB at F (see fig. 53). What is the relation between ∠ BOF and ∠ BAC. Give reasons.

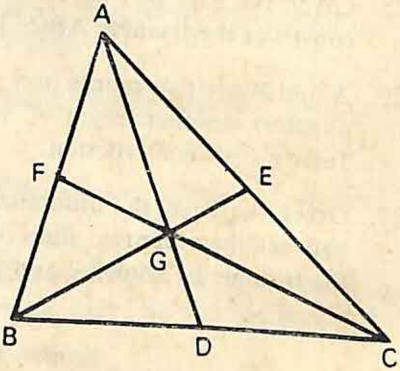


Fig. 53.

9. "In the triangle ABC, angle BAC is a right angle. The point of intersection of perpendicular bisectors of AB and AC will intersect within the triangle". Is the above statement correct ? If not, give the correct statement.

10. In fig. 54, AD is perpendicular to BC. State which of the two sides AB and BD is greater than the other. State also which of the two sides AC and CD is greater than the other. Give reasons in each case.

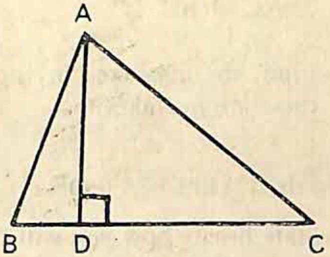


Fig. 54.

11. BE and CF are two medians of a triangle ABC. Suggest how a line is to be drawn through A so that it bisects BC, without using a compass. Justify your answer.

12. AB is a line segment. Through B, another line is drawn and segments of equal lengths BC and BD are cut off on opposite sides of B. AC and AD are joined. The median CR of $\triangle$ ACD intersects AB at O. State, with reasons, whether the following statement is true : "O is the point of trisection of the segment AB".

13. Utilising the result that in any triangle, sides opposite to equal angles are equal to one another, one proceeds wrongly to prove that three sides of a right-angled triangle are equal to one another as follows : ABC is a right-angled triangle and $\angle$ ABC = 90°. BD is drawn perpendicular to AC (see fig. 55). Now, since $\angle$ BDA = $\angle$ BDC = 90° and AB and BC are sides opposite to these equal angles, $\therefore$ AB = BC. Again, since $\angle$ ABC = $\angle$ BDC = 90° and AC and BC are sides opposite to these equal angles,

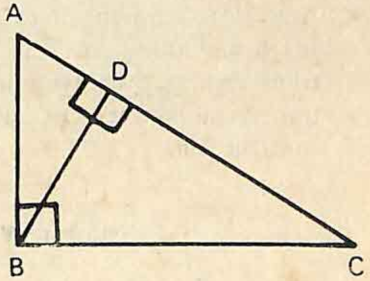


Fig. 55.

$\therefore AC = BC$

$\therefore AB = BC = AC.$

Find the mistakes in the above proof and state the reason(s) why these are mistakes.

Objective : APPLICATION

14. State briefly how you will construct a triangle ABC having given the side BC, $\angle B$ and $AB - AC$.

15. The side BC of a triangle ABC is produced to D such that $\angle ACD$, the exterior angle of $\angle ACB$ is 70°. State, with reasons, whether AB and AC will be equal or not. If not, which is greater ?

16. P,B and C are three points in a plane such that P lies within $\angle EDC$, where D is the middle point of BC and $ED \perp BC$. Two persons start from P at the same time and walk along PB and PC with the same speed. Prove that the person who reaches B will take more time than the person who reaches C.

Objective : SKILL

17. You are given an angle of 45°. State the procedure of dividing this angle into 3 equal parts with the help of a compass and a ruler.

18. Any three straight lines AO, BO and CO meet at O such that the line BO lies within $\angle AOC$. Take any point L on BO. Through L, a transversal of the three lines is to be drawn such that the segment of the transversal between OA and OC is bisected at L. State the procedure of construction.

FORM : **Very Short Answer Type**

Objective : KNOWLEDGE

1. ABC is a triangle. E is the mid-point of AC. P divides BE such that

BP : PE = 2 : 1. What special name has P with reference to the triangle ABC ?

2. ABC is a triangle. The angular bisectors of angles A and B meet at O. OE and OF are perpendiculars to BC and AC cutting them at E and F respectively. If OE is x cm., what is the length of OF ?

3. What is the name of the point where two altitudes of a triangle meet ?

Objective : UNDERSTANDING

4. In the triangle ABC, AB = AC.
   (i) Can ∠ A be a right angle ?
   (ii) Can ∠ B be a right angle ?
   (iii) Can ∠ C be an obtuse angle ?

5. ABC is any triangle and D is a point on BC such that ∠ ADC is obtuse. Is it always ture that AC > AB ?

Objective : APPLICATION

6. Through the vertices A,B,C of the △ ABC, three lines are drawn such that each line through a vertex is parallel to the side opposite to that vertex. Let the triangle, thus formed, be denoted by △ PQR. Through P,Q,R, three more straight lines are drawn such that each line through each vertex of △ PQR is parallel to the side of △ PQR, opposite to that vertex. Let us denote the triangle, thus obtained, by △ UVW. (i) Will the perpendicular bisectors of the sides of △ UVW always pass through the orthocentre of △ ABC ? (ii) Will the perpendicular bisectors of the sides of △ PQR always pass through the orthocentre of △ ABC ?

Objective : SKILL

7. A frog is jumping along the three sides of a triangle. In each jump, it traverses a distance of 5 cm. of the straight path along each side. It had to jump 4 times to traverse each side of the triangle completely. Will the angles subtended at the circumcentre of the triangle by the line segments

traversed by the frog in successive jumps be equal ? How much angle will it describe at the above circumcentre in two jumps ?

FORM : **Objective Type**

(i) *Multiple choice type* :

In each of the questions (No. 1 to 7), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right hand margin.

Objective : KNOWLEDGE

1. If PQR is a triangle in which $\angle R > \angle Q$, then
   (A) PQ = PR
   (B) PQ > PR
   (C) PQ < PR
   (D) nothing can be said about PQ and PR ( )

2. An orthocentre is a point where
   (A) the bisectors of three angles meet
   (B) the perpendicular bisectors of three sides meet
   (C) the three perpendiculars drawn from the vertices to the opposite sides meet
   (D) the three medians meet ( )

Objective : UNDERSTANDING

3. ABC is an equilateral triangle and I is its incentre. IX is drawn perpendicular to AC cutting AC at X. Then the ratio of the angles IAX, AIX and AXI of $\triangle$ AIX is
   (A) 2 : 1 : 3
   (B) 1 : 2 : 3
   (C) 1 : 2 : 1
   (D) none of the above ( )

4. ABC is any triangle such that $\angle A = x^\circ$, $\angle B = y^\circ$ and $\angle C = z,^\circ$ where $x^\circ + y^\circ + z^\circ = 180^\circ$. L,M,N are points on BC, CA and AB respectively and are such that $AL \perp BC$, $BM \perp AC$ and $CN \perp AB$. Then the angles LOC, MOC and MOA will be respectively
   (A) z°, x° and y°
   (B) z°, y° and x°
   (C) x°, y° and z°
   (D) y°, x° and z° ( )

Objective : APPLICATION

5. ABC is an equilateral triangle and I is its in-centre. Perpendiculars ID, IE and IF are drawn to the three sides of the triangle. Then the minor arc DE of the in-circle of the triangle ABC is
   (A) a quadrant of the in-circle
   (B) half of the in-circle
   (C) $\frac{1}{3}$ of the in-circle
   (D) $\frac{1}{6}$ of the in-circle ( )

6. The two diagonals of a rhombus ABCD intersect at O. One circle is drawn through the points A,O and B and another circle is drawn through the points D, O and C. If r be the side of the rhombus, then the total length of the arcs AOB and DOC is
   (A) πr
   (B) $2\pi r$
   (C) less than $2\pi r$
   (D) greater than $2\pi r$ ( )

Objective : SKILL

7. The base, the difference between the base angles and the sum of two other sides of a scalene triangle are given. With these data, we can draw
   (A) one and only one triangle
   (B) two triangles
   (C) more than two triangles
   (D) no triangle at all ( )

(ii) *True-false type* :

In the following statements marked with 1,2,3, etc., write "T" against each true statement and "F" against each false statement.

Objective : KNOWLEDGE

1. In fig. 56, OB = OE and BE is the median. Then O is the centroid of the triangle.

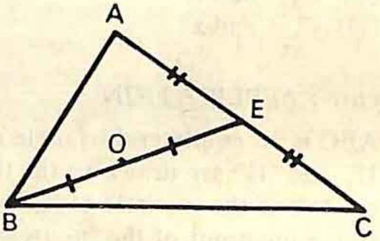


Fig. 56.

2. The point of intersection of the perpendicular bisectors of the sides of a triangle is called the in-centre of the inscribed circle of the triangle.

Objective : UNDERSTANDING

3. The orthocentre of an obtuse-angled triangle lies outside the triangle.
4. The orthocentre of a right-angled triangle is the vertex of the triangle which lies in the right angle.
5. In an isosceles triangle, the medians and the perpendicular bisectors coincide.
6. In an equilateral triangle, the circumcentre and the centroid coincide.
7. In a $\triangle$ ABC, AB = 8 cm., BC = 4 cm., and CA = 5 cm. Then $\angle$ C is the greatest angle.
8. The circumcentre of a right-angled triangle lies on its hypotenuse.
9. The locus of a point which is at a constant distance from a straight line is a circle.

10. A circle described on the hypotenuse of a right-angled triangle as a diameter will pass through the opposite vertex.

11. In fig. 57, triangles ABC and ACD have the same circumcentre. Then ABCD is a cyclic quadrilateral.

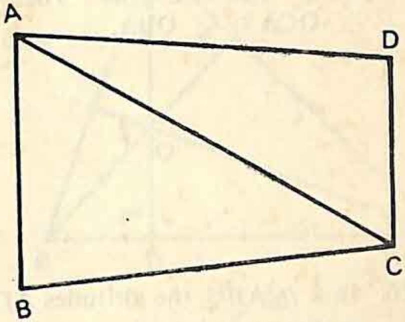


Fig. 57.

12. I is the in-centre of the scalene triangle ABC. AI meets BC at P. Then the radius of the in-circle is IP.

13. In fig. 58, the perpendicular bisector of PQ will divide the arc PSQ in the ratio 2 : 1.

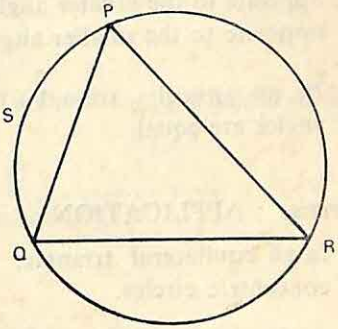


Fig. 58.

14. The angular bisector of $\angle X$ and the perpendicular bisector of XZ of an equilateral triangle will meet on the circumcircle of the $\triangle$ XYZ.

15. In fig. 59, AB = AC and OC > OB, where O is any point within △ ABC. Then ∠ OCA > ∠ OBA.

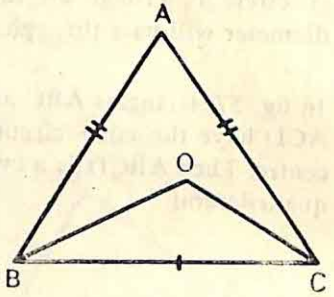


Fig. 59.

16. In a △ ABC, the altitudes AD and BE meet at O. Then OECD O is a cyclic quadrilateral.

17. ABCD is a rhombus whose diagonals cut at O. All circles drawn on AB, BC, CD and DA as diameters will pass through O.

18. In two triangles ABC and DEF, ∠ A > ∠ D. Hence BC which is opposite to the greater angle will always be greater than EF which is opposite to the smaller angle.

19. In an isosceles triangle, two medians drawn from the vertices of equal angles are equal.

Objective : APPLICATION

20. In an equilateral triangle, the in-circle and the circumcircle will be concentric circles.

21. S is the circumcentre of the right-angled triangle ABC. G is its centroid. Then B, G, S will be collinear.

(iii) *Sentence completion type* :

Fill up the blanks with appropriate answers given in the brackets :

22. In fig. 60, ABC is a scalene triangle. AG : GD = BG : GE = CG : GF = 2 : 1. Therefore, a circle with centre G will pass through D, E, F.

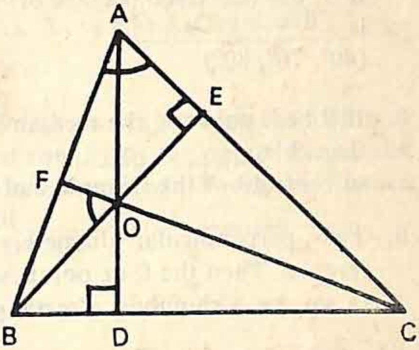


Fig. 60.

Objective : KNOWLEDGE

1. The point of concurrence of the medians of a triangle is called ______ .
   (circumcentre, centroid, orthocentre, in-centre)
2. G is the centroid of the $\triangle$ ABC. If the median AD = 6 cm., then the length of AG is______________.
   (2 cm., 4 cm., 3 cm.)
3. S is the circumcentre of $\triangle$ ABC. If AS = 4 cm., the length of BS is____.
   (8 cm., 2 cm., 4 cm.)
4. In $\triangle$ ABC, AB < AC. Then $\angle$ C is ____________.
   (an obtuse angle, a right angle, an acute angle)

Objective : UNDERSTANDING

5. P is any point within the $\triangle$ ABC such that PQ and PR drawn perpendicular to AB and AC respectively are equal. Then the locus of P is______.
   (a median through A, an altitude through A, an angular bisector of $\angle$ A)
6. If I be the in-centre of a $\triangle$ ABC, then $\angle$ AIB is ____________ .
   $\left(\frac{A + B}{2}, 180^\circ - (A + B), 180^\circ - \frac{A + B}{2}\right)$

7. If S be the circumcentre of △ ABC and ∠ ASB = 140°, then ∠ C is __________________.
(40°, 70°, 80°)

8. If S be a point on the median PT of △ PQR such that TS : SP = 2 : 1, then S is______________.
(a centroid of the triangle, not a centroid of the triangle)

9. Two perpendicular diameters of a circle meet at points on the circumference. Then the four points will be the vertices of ______________.
(a square, a rhombus, a rectangle)

Objective : APPLICATION

10. ∠ PSR is a right angle. D is the mid-point of SR. The angular bisector of ∠ PSR and perpendicular bisector of SR meet at O. Then △ OSD is ________________.
(scalene, right-angled with unequal sides, isosceles right-angled)

11. D, E, F are the mid-points of the three sides of △ ABC. The orthocentre of △ DEF is the__________________. of △ ABC.
(centroid, orthocentre, circumcentre)

12. PQSR is a rhombus. Circles are drawn with PR and QS as diameters. Then the circles will_________________.
(cut at two points, touch at one point, not cut at all)

13. The circle drawn with one of the equal sides of an isosceles triangle as a diameter will pass through a point on the base of the triangle, which divides the base in the ratio ____________________ .
(1 : 2, 1 : 1)

Objective : SKILL

14. ABC is an isosceles triangle and AB = AC. With A as the centre and AB as the radius, we draw a circle. We now divide BC into 5 equal parts

by means of the points A_1, A_2, A_3 and A_4. Then the angles subtended by the equal segments BA_1, A_1A_2, A_2A_3, A_3A_4 and A_4C at A will_____ .
(be equal, not be equal)

15. With O as centre, we draw a circle which passes through two points A and B. Arc on the chord AB is divided equally by the points A_1, A_2 and A_3. A_1O, A_2O and A_3O are joined. Then the chord AB will be divided by these lines into four parts which will__________________ .
(be equal, not be equal)

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CHAPTER XIV

A MODEL OF A QUESTION PAPER*

(Before starting, read the article in chapter III)

Design of the question paper

(a) Distribution of marks according to objectives and contents.

Objectives	*Total marks in*				*Grand Total*
	Algebra	*Geometry*	*Arithmetic*	*Trigonometry*	
Knowledge (K)	8	8	4	4	24
Understanding (U)	19	19	8	9	55
Application (A)	7	7	3	2	19
Skill (S)	1	1	—	—	2
Total	35	35	15	15	100

---

*The percentages of weightages to different objectives, forms of questions and contents for this question paper have been chosen arbitrarily and are not based on any policy decision. The purpose of this model question paper is simply to let a questions-setter know the systematic procedure of framing a question paper,

(b) **Distribution of marks according to form of questions.**

Form of questions	*Total marks in*				*Grand Total*
	Algebra	*Geometry*	*Arithmetic*	*Trigonometry*	
Objective type (O)	10	10	2	8	30
Very short answer type (VSA)	6	6	—	2	14
Short answer type (S.A.)	14	14	9	5	42
Essay type (E)	5	5	4	—	14
Total	35	35	15	15	100

(c) **Number of questions according to objectives and contents.**

Objectives	*Total number of questions in*				*Grand Total*
	Algebra	*Geometry*	*Arithmetic*	*Trigonometry*	
Knowledge (K)	6	6	2	4	18
Understanding (U)	10	10	3	6	29
Application (A)	4	4	1	2	11
Skill (S)	1	1	—	—	2
Total	21	21	6	12	60

(d) **Number of questions according to form of questions and contents.**

Form of questions	*Total number of questions in*				*Grand Total*
	Algebra	*Geometry*	*Arithmetic*	*Trigonometry*	
Objective type (O)	10	10	2	8	30
Very short answer type (VSA)	6	6	—	2	14
Short answer type (SA)	4	4	3	2	13
Essay type (E)	1	1	1	—	3
Total	21	21	6	12	60

(e) **Distribution of marks and estimated maximum time for different forms of questions.**

Form	*No. of questions*	*Marks*	*Estimated time*
O	30	30	48 mins.
VSA	14	14	20 mins.
SA	13	42	67 mins.
E	3	14	45 mins.
Total	60	100	180 mins.

(f) Distribution of marks and number of questions of differeut objectives.

Objectives	*No. of questions*	*Marks*
K	18	24
U	29	55
A	11	19
S	2	2
Total	60	100

O	stands for objective type.	K	stands for knowledge.
VSA	stands for very shot answer type.	U	stands for understanding.
SA	stands for short answer type.	A	stands for application.
E	stands for essay type.	S	stands for skill.

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Objective → ← Contents	Units ↓ Form of questions →	Knowledge: Essay type	Knowledge: Short answer type	Knowledge: Very short answer type	Knowledge: Objective type	Understanding: Essay type	Understanding: Short answer type	Understanding: Very Short answer type	Understanding: Objective type
Algebra	U—I							1 (1)	1 (1)
	U—II			1 (1)	1 (1)		1 (3)	1 (1)	1 (1)
	U—III		1 (3)	1 (1)	1 (1)	1 (5)	1 (4)	1 (1)	1 (1)
	U—IV				1 (1)				1 (1)
Arithmetic	U—V		1 (3)		1 (1)	1 (4)	1 (3)		1 (1)
Trigonometry	U—VI			1 (1)	3 (3)		2 (5)	1 (1)	3 (3)
Geometry	U—VII				1 (1)			1 (1)	1 (1)
	U—VIII			1 (1)	1 (1)		1 (4)	1 (1)	1 (1)
	U—IX		1 (3)	1 (1)	1 (1)		1 (3)		1 (1)
	U—X					1 (5)		1 (1)	1 (1)
Total			3 (9)	5 (5)	10 (10)	3 (14)	7 (22)	7 (7)	12 (12)

Note : Figures outside the brackets indicate the number of questions and

THE QUESTION PAPER

Application				Skill				Total
Essay type	*Short answer type*	*Very short answer type*	*Objective type*	*Essay type*	*Short answer type*	*Very short answer type*	*Objective type*	
							1 (1)	3 (3)
								5 (7)
	1 (4)	1 (1)	1 (1)					10 (22)
			1 (1)					3 (3)
	1 (3)							6 (15)
			2 (2)					12 (15)
							1 (1)	4 (4)
		1 (1)	1 (1)					7 (10)
	1 (4)		1 (1)					7 (14)
								3 (7)
	3 (11)	2 (2)	6 (6)				2 (2)	60 (100)

those within the brackets indicate total marks.

QUESTION PAPER

Class IX

Subject : **Mathematics**

Time : 3 hours

Maximum Marks : 100

All questions are compulsory.

Section A

In each of the following questions (No. 1 to 15), there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against the correct answer should be written in bold letter in the bracket provided in the right hand margin.

1. If antilog $p = x$ and antilog $q = y$, then $\log (xy)$ is equal to
   - (A) pq
   - (B) p^q
   - (C) q^p
   - (D) $p + q$

( )

2. If $n(A) = p$ and $n(B) = q$, then $n(A \times B)$ is equal to
   - (A) pq
   - (B) $p + q$
   - (C) $p + q + 1$
   - (D) none of the above

( )

3. The sum of all the internal angles of a regular hexagon each of whose sides is 8 cm. is.
   (A) 120°
   (B) 720°
   (C) 960°
   (D) none of the above ( )

4. A circle circumscribes a scalene triangle and one side of the triangle passes through the centre of the circle. Then the triangle must be
   (A) a right-angled triangle with two sides equal
   (B) a right-angled triangle with two sides unequal
   (C) an obtuse-angled triangle
   (D) an acute-angled triangle ( )

5. Adding x% of y to y is the same as multiplying y by
   (A) $\frac{x}{100}$
   (B) $x\left(\frac{1}{100}+1\right)$
   (C) $\frac{x}{100}+1$
   (D) none of the above ( )

6. $1-\cos^2 60°$ is equal to
   (A) $\cos^2 30°$
   (B) $\sin 60°$
   (C) $-\sin^2 60°$
   (D) none of the above ( )

7. The set { 10, 15, 21, 28 } can be expressed as
   (A) $\{ x \mid 10 \leqslant x \leqslant 28 \}$
   (B) $\{x \mid x = \frac{p(p+1)}{2}, p \in N \text{ and } 4 \leqslant p \leqslant 7\}$
   (C) $\{ x \mid 2x+1 = 21 \}$
   (D) none of the above ( )

8. If $2x + 3y \leqslant 6, y \geqslant 0, x \geqslant 0$, then one of the solutions is
(A) $x = -2, y = 3$
(B) $x = 1, y = 2$
(C) $x = 1, y = 1$
(D) $x = 1, y = -2$
( )

9. If $\log \sqrt{a} = r$, then $\sqrt[2r]{a}$ is equal to
(A) $\sqrt{10}$
(B) 100
(C) 10
(D) none of the above
( )

10. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is
(A) a right-angled triangle
(B) an equilateral triangle
(C) an obtuse-angled triangle
(D) an acute-angled triangle other than an equilateral triangle ( )

11. PQ and RS are equal chords of a circle. The perpendicular bisectors of PQ and RS will
(A) intersect inside the circle
(B) intersect outside the circle
(C) intersect on the circle
(D) never intersect
( )

12. $\sin^4 \theta + \cos^4 \theta$ is the same as
(A) $\sin^2 \theta^2 + \cos^2 \theta^2$
(B) $\sin \theta^4 + \cos \theta^4$
(C) $(\sin \theta + \cos \theta)^4$
(D) none of the above
( )

13. H,K are the mid-points of the sides AB and AC of $\triangle$ ABC and O, any point on BC is joined to A. If L,M are the mid-points of OB, and OC respectively, then HKML is always

(A) a square
(B) a rectangle
(C) a rhombus
(D) any other parallelogram ( )

14. If the two adjacent sides of a rectangle be of magnitudes $\sqrt{\cos\theta}$ and $\sqrt{\sin\theta}$, then the square of its diagonal will always be
(A) equal to $\sqrt{2}$
(B) less than $\sqrt{2}$
(C) greater than $\sqrt{2}$
(D) less than or equal to $\sqrt{2}$ ( )

15. If $x = 2 + 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$, then $x^3 - 6x^2 + 3x$ is
(A) 2
(B) 0
(C) $3^{\frac{1}{3}} + 3^{\frac{2}{3}}$
(D) none of the above ( )

In the following statements (No. 16 to 20), write "T" against each true and "F" against each false statements at the end of each statement :

16. If a is a real positive number and θ is acute, then $\sin\theta = 1 + a$ must hold good.

17. The square of an irrational number is always a rational number.

18. $\cos^2\theta + \cos^2(90^\circ - \theta)$ is constant for all values of θ.

19. If $y^{\frac{1}{q}} = z^{\frac{1}{r}}$, where q and r are known quantities and $\frac{x}{y} = \frac{y}{z} = 2$, then $\log_{10} x + \log_{10} y + \log_{10} z$ can be expreesed in terms of the known quantities.

20. If $0 \leqslant \cos\theta < \frac{1}{2}$, then $\sin\theta \geqslant \frac{1.5}{\sqrt{3}}$.

21. Given that $\triangle ABW \cong \triangle MKF$ and both the triangles are scalene. In column I, elements of $\triangle ABW$ and in column II, those of $\triangle MKF$ are given. Match the elements of column I with those in column II so that the two elements are equal, in the manner like (N) – (X), where N is a positive integer and X is an English alphabet.

Column I

(1) AB
(2) $\angle B$
(3) BW
(4) $\angle W$
(5) $\angle A$
(6) AW

Column II

(A) $\angle M$
(B) $\angle F$
(C) MF
(D) KF
(E) MK
(F) $\angle K$

Fill up the blanks with appropriate answers given in the brackets in the following (No. 22 to 30) :

22. If $a > b$, then $-a$ ______________ $-b$.
$(>, <, =)$

23. In a right-angled triangle ABC, the hypotenuse $AB = 1$, $BC = x$ and $AC = y$. Then $\cos \angle ABC =$ ______________.
$(1, x, y, \frac{x}{y})$

24. If a parallelogram with unequal adjacent sides has one right angle, then it is a ______________.
(square, rectangle, rhombus)

25. If I be the in-centre of $\triangle ABC$, then $\angle AIB$ is ______________.
$\left[\frac{A+B}{2},\ 180^\circ - (A+B),\ 180^\circ - \frac{A+B}{2} \right]$

26. If n be the number of years, r% be the rate of interest and the simple interest $= \frac{P \times n \times r}{50}$, then the principal is ______________.
$(P, 2P, \frac{P}{2})$

27. If two equal sides of an isosceles triangle be sin θ and cos θ, then the value of θ is ____________________ .
(30°, 45°, 60°)

28. In the region represented by $3x + 7y \leqslant 21$, $x \leqslant 3$, $y \geqslant 0$, the line $12x + 7y = 48$ will pass through ________________ point (s) of the region.
(one, two, more than two)

29. If the two internal bisectors of two opposite angles of a cyclic quadrilateral intersect the circumference at Q and R, then QR will be ____________________ .
(a diameter, a chord other than a diameter)

30. There are six persons standing at six points O,A,B,C, D and E such that no three points are collinear. Each person wishes to go to the next consecutive point on his right in the shortest path i.e. for example, the person at O wishes to go to A along OA, that at A to B along AB and so on. Assume that each person walks with the same constant speed u metres/sec. They started at the same time and reached their destinations at the same time after t secs. Then the angle between the direction of one person and the next person on his right will be ________________ .
(120°, 60°, 30°).

31. For any set A, are the sets $A \cap A$ and $A \cup A$ the same ?

32. Write the co-ordinates of the point where the lines $x = -1$ and $y = 0$ intersect.

33. If the two diagonals of a parallelogram ABCD are equal, then what is the value of $\angle$ ABC ?

34. ABC is a triangle. The perpendicular bisectors of AB and BC meet at O. Write down the ratio OA : OB : OC.

35. Given that $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$. What is the value of cos 72° ?

36. For what values of a number x ($\neq 0$), its reciprocal will be less than it ?

37. If A = { a } and B = { b, c }, write down a relation between a, b, c, such that A = B.

38. What is the largest positive integer not in the set $\{x \mid x > 2\}$?

39. In a triangle ABC, a line LM is drawn through A parallel to BC. What is the difference between the angles CAM and BAL in terms of angles B and C ?

40. A right-angled triangle has two equal sides. Write down the ratio of its angles.

41. ABC is any triangle and D is a point on BC such that $\angle$ ADC is obtuse. Is it always true that AC > AB ?

42. Write an acute angle for which $\sin \phi = \cos \phi$.

43. Of all the values of x and y satisfying both $x + y \geqslant 4$ and $x - y = 0$, what are the least values of x and y ?

44. Is it possible to construct a quadrilateral which has three obtuse exterior angles ?

Section B

45. Represent (i) $x \leqslant 3$ and (ii) $x \geqslant -1$ on the number line and shade the common part of the above two regions.

46. "The angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the circumference." Is the above statement correct ? If not, give the correct statement.

47. 6 pens are bought at Rs 5 and 5 pens are sold at Rs 6. What is the gain or loss in per cent ?

48. If $A = \{ x \mid x \text{ is a positive even integer and } x < 100 \}$,
$B = \{ x \mid x \text{ is a multiple of 5 and } x < 110 \}$
and $C = \{ x \mid x \in N \text{ and } 25 < x \leqslant 100 \}$,
find (i) $(A \cap B) \cap C$ (ii) $A \cap (B \cap C)$.

49. Solve the following equations :
(i) $(3x + 1)(2x - 4) - x(6x - 7) = 0$
(ii) $(x + 1)^2 - (x - 1)^2 = 3$.

50. In $\triangle$ PQR, PQ = QR. L, M, N are respectively the middle points of the sides PQ, QR and RP. Prove that LN = MN and $\angle$ PLM = $\angle$ PNM.

51. Can you draw two distinct circles—one through the points A,B,C,D and the other through the points A, B, D, E ? Give reasons in support of your answer.

52. The balance in a man's account in a bank was Rs. 500 on the 1st January of a year. He deposited Rs. 300, Rs. 2000 and Rs. 2400 on January 7, January 10 and January 12 respectively and he withdrew Rs. 300 and Rs. 2400 on January 4 and January 11 respectively. If the rate of interest be 6% per annum and if the interest is paid on the minimum balance from the 10th to the end of a month, then calculate the man's interest for the month of January.

53. Prove that $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$.

54. If $\sin\theta = \dfrac{m^2 - n^2}{m^2 + n^2} (m > n)$, find $\cos\theta$ in the simplest form.

55. Find the solution of :
$2x - y > 3$
and $5x - 2y = 8$.

56. ABC is an isosceles right-angled triangle, right-angled at B. O is the mid-point of AC. Prove that the line joining the circumcentres of the two triangles AOB and BOC will bisect OB at right angles.

57. A reduction of 20% in the price of sugar enables me to get 15 Kg. more for Rs. 240. What is the reduced price ?

58. If unity is subtracted from the numerator x of a fraction and 2 is added to the denominator y, the fraction becomes $\frac{1}{3}$ If unity is added to x and 12 is added to y, the fraction becomes $\frac{1}{4}$ Find the fraction.

59. Prove that the sum of any two sides of a triangle is greater than twice the median which bisects the third side.

60. On what sum will the difference between the simple and compound interests for 3 years at 5% per annum amount to Rs. 36.60 ?

SCORING KEY AND MARKING SCHEME

1. D
2. A
3. B
4. D
5. C
6. A
7. B
8. C
9. C
10. A
11. A
12. D
13. D
14. D
15. A
16. F
17. F
18. T
19. T
20. T
21. (1) – (E), (2) – (F), (3) – (D), (4) – (B), (5) – (A), (6) – (C).
22. $<$
23. x
24. rectangle
25. $180° - \frac{A + B}{2}$
26. 2p
27. 45°
28. one
29. a diameter
30. 60°
31. Yes
32. $(-1, 0)$
33. 90°
34. 1 : 1 : 1
35. $\frac{\sqrt{5} - 1}{4}$
36. $x > 1$ or $-1 < x < 0$
37. a = b = ;
38. 2
39. $\angle B - \angle C$ or $\angle C - \angle B$
40. 1 : 1 : 2
41. No
42. 45°
43. 2, 2
44. Yes

	Marks allotted
45. (i) To draw the figure showing $x \leqslant 3$	1
To draw the figure showing $x \geqslant -1$	1
(ii) To draw the figure showing the common region	1
Total	3
46. No.	1
The correct statement is : "The angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the remaining part of the circumference".	2
Total	3
47. Gain in 1 pen = Rs $(\frac{6}{5} - \frac{5}{6})$ = Rs $\frac{11}{30}$	1
In Rs $\frac{5}{6}$, the gain is Rs $\frac{11}{30}$ ∴ In Rs 100, the gain is Rs $(\frac{11}{30} \times \frac{6}{5} \times 100)$ = Rs 44.	2
Total	3
48. $A = \{ 2, 4, 6, 8, \ldots 98 \}$ $B = \{ 5, 10, 15, \ldots 105 \}$ $C = \{ 26, 27, 28, \ldots 100 \}$ (i) $A \cap B = \{ 10, 20, 30, \ldots 90 \}$	
$(A \cap B) \cap C = \{ 30, 40, 50, 60, 70, 80, 90 \}$	1
(ii) $A \cap (B \cap C) = (A \cap B) \cap C = \{ 30, 40, 50, 60, 70, 80, 90 \}$	1
Total	3

49. (i) $6x^2 - 10x - 4 - 6x^2 + 7x = 0$ 1

or, $-3x - 4 = 0 \therefore x = -\frac{4}{3}$ 1

(ii) $x^2 + 2x + 1 - x^2 + 2x - 1 = 3$ 1

or, $4x = 3 \therefore x = \frac{3}{4}$ 1

Total 4

50. $\left.\begin{array}{l} LN = \frac{1}{2}.\ QR \\ MN = \frac{1}{2}.\ PQ \therefore LN = MN \end{array}\right\}$ 2

Again, $LM \parallel PR$ and $MN \parallel PQ$.

$\left.\begin{array}{l} \text{Also } LM = \frac{1}{2}.\ PR = PN \text{ and } MN = \frac{1}{2}.\ PQ = PL \\ \therefore LMNP \text{ is a parallelogram. } \therefore \angle PLM = \angle PNM. \end{array}\right\}$ 2

Total 4

51. No. 1

We know that through any three non-collinear points, one and only one circle can pass. If one circle passes through A, B, C, D and a different circle passes through A, B, D, E, then through the three points A, B and D, two distinct circles will pass, which is not possible. Hence, only one circle can pass through the points A, B, C, D as well as A, B, D, E. 2

Total 3

52. Balances on Jan. 4, Jan. 7, Jan. 10, Jan. 11 and Jan. 12 are respectively Rs 200, Rs 500, Rs 2500, Rs 100, Rs 2500. 1

Minimum balance for Jan. 10 to Jan. 31 = Rs 100. 1

Interest for January $= \frac{\text{Rs } 6}{12} = \text{Re } 0.50.$ 1

Total 3

53. $\left.\begin{aligned} \text{L.H.S.} &= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta \\ &= 1 - 2\sin^2\theta\cos^2\theta \end{aligned}\right\}$ 2

54. $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1 - \left(\frac{m^2-n^2}{m^2+n^2}\right)^2}$ 1

$= \sqrt{\frac{(m^2+n^2)^2-(m^2-n^2)^2}{(m^2+n^2)^2}}$

$= \frac{2\,mn}{m^2+n^2}$ 2

Total 3

55. $\left.\begin{array}{l} \text{From the equation, we have } y = \frac{5x-8}{2} \\ \therefore \text{ From the ineqnation,} \\ 2x - \frac{5x-8}{2} > 3 \\ \text{or, } 4x - 5x + 8 > 6 \\ \text{or, } -x > -2 \;\therefore\; x < 2 \end{array}\right\}$ 2

Now, from the given equation, $x = \frac{8+2y}{5} < 2$

$\Rightarrow y < 1$ 1

$\therefore$ Solution is $x < 2$ and $y < 1$ such that $5x - 2y = 8$ 1

Total 4

56. △ BOC and △ OBA are congruent (details to be shown) and hence ∠ BOC = ∠ BOA = 1rt. angle. 2

∴ The circumcentres of △ BOC and △ OBA will lie on BC and AB respectively at their middle points. 1

Hence, the line joining these two points will bisect BO perpendicularly (details to be shown). 1

Total 4

57. In Rs 100, the gain is Rs 20.

$\therefore$ In Rs 240, the gain is Rs $\frac{20}{100} \times 240 =$ Rs 48. 2

$\therefore$ The price of 15 kg. of sugar at the reduced price = Rs 48.
$\therefore$ The price of 1 kg. of sugar at the reduced price
$= \frac{\text{Rs } 48}{15} =$ Rs 3.20. 2

Total 4

58. $$\left.\begin{aligned} \frac{x-1}{y+2} &= \frac{1}{3} \\ \frac{x+1}{y+2} &= \frac{1}{4} \end{aligned}\right\} \quad 2$$

Solving, we get $x = 3$, $y = 4$.
$\therefore$ The required fraction $= \frac{3}{4}$. 3

Total 5

59. Produce the median AD to E such that AD = DE.
Join EC. $\triangle$ ABD and $\triangle$ DEC are congruent
(details to be shown).
$\therefore$ AB = EC. 3

From $\triangle$ ACE, AC + CE > AE
or, AC + AB > 2 AD. 2

Total 5

60. $$\left.\begin{aligned} I_1 &= \frac{P\,r\,t}{100} = \frac{3P}{20} \\ I_2 &= P\left[\left(1 + \frac{1}{20}\right)^3 - 1\right] \end{aligned}\right\} \quad 2$$

$$\left.\begin{aligned} &\therefore \frac{3P}{20} - P\left[\frac{21^3}{20^3} - 1\right] = 36.60 \\ &\therefore P = \text{Rs } 4800. \end{aligned}\right\} \quad 2$$

Total 4

Question-wise Analysis

Section A

Question No.	*Objective*	*Form of Questions*	*Content Units*	*Marks Allotted*	*Estimated maximum time (in minutes) for Answering*
1.	K	O	Unit-IV	1	1
2.	K	O	Unit-II	1	1
3.	K	O	Unit-VII	1	1
4.	K	O	Unit-IX	1	1
5.	K	O	Unit-V	1	1
6.	K	O	Unit-VI	1	1
7.	U	O	Unit-II	1	1
8.	U	O	Unit-III	1	2
9.	U	O	Unit-IV	1	2
10.	U	O	Unit-VII	1	1
11.	U	O	Unit-IX	1	2
12.	U	O	Unit-VI	1	1
13.	A	O	Unit-VIII	1	2
14.	A	O	Unit-VI	1	2
15.	S	O	Unit-I	1	3

Section A—*Contd.*

Question No.	*Objective*	*Form of Questions*	*Content Units*	*Marks Allotted*	*Estimated maximum time (in minutes) for Answering*
16.	K	O	Unit-VI	1	1
17.	U	O	Unit-I	1	1
18.	U	O	Unit-VI	1	1
19.	A	O	Unit-IV	1	2
20.	A	O	Unit-VI	1	2
21.	K	O	Unit-VIII	1	3
22.	K	O	Unit-III	1	1
23.	K	O	Unit-VI	1	2
24.	U	O	Unit-VIII	1	1
25.	U	O	Unit-X	1	2
26.	U	O	Unit-V	1	1
27.	U	O	Unit-VI	1	2
28.	A	O	Unit-III	1	2
29.	A	O	Unit-IX	1	2
30.	S	O	Unit-VII	1	3
31.	K	VSA	Unit-II	1	1
32.	K	VSA	Unit-III	1	1
33.	K	VSA	Unit-VIII	1	1
34.	K	VSA	Unit-IX	1	1

Section A—*Contd.*

Question No.	*Objective*	*Form of Qestions*	*Content Units*	*Marks Allotted*	*Estimated maximum time (in minutes) for Answering*
35.	K	VSA	Unit-VI	1	1
36.	U	VSA	Unit-I	1	2
37.	U	VSA	Unit-II	1	1
38.	U	VSA	Unit-III	1	1
39.	U	VSA	Unit-VII	1	2
40.	U	VSA	Unit-VIII	1	2
41.	U	VSA	Unit-X	1	2
42.	U	VSA	Unit-VI	1	1
43.	A	VSA	Unit-III	1	2
44.	A	VSA	Unit-VIII	1	2
45.	K	SA	Unit-III	3	4
46.	K	SA	Unit-IX	3	5
47.	K	SA	Unit-V	3	5
48.	U	SA	Unit-II	3	5
49.	U	SA	Unit-III	4	6
50.	U	SA	Unit-VIII	4	6
51.	U	SA	Unit-IX	3	5
52.	U	SA	Unit-V	3	5
53.	U	SA	Unit-VI	2	4
54.	U	SA	Unit-VI	3	5

Section A—*Contd.*

Question No.	*Objective*	*Form of Questions*	*Content Units*	*Marks Allotted*	*Estimated maximum time (in mimutes) for Answering*
55.	A	SA	Unit-III	4	6
56.	A	SA	Unit-IX	4	6
57.	A	SA	Unit-V	3	5
58.	U	E	Unit-III	5	15
59.	U	E	Unit-X	5	15
60.	U	E	Unit-V	4	15
				Total	180

Note : K stands for Knowledge.
U stands for Understanding.
A stands for Application.
S stands for Skill.
O stands for Objective type.
VSA stands for Very short answer type.
SA stands for Short answer type.
E stands for Essay type.

ANSWERS

UNIT I

FORM : **Essay Type**

1. Yes, 2. 2. Yes. 5. Yes.

6. (i) 15 (ii) Yes (iii) No (iv) No. 7. $\frac{a}{b}$ or $\frac{c}{d}$

9. (i) 4 (ii) No (iii) Yes 10. $\frac{4}{3}$.

FORM : **Short Answer Type**

1. 3 and 5 or 4 and 7 are co-prime.

2. (a) Yes (b) Yes (c) Yes. 3. (i) -1 (ii) -1.

5. (i) 4, $\sqrt{8\sqrt{2}}$ (ii) No.

6. Mistakes lie in steps 3 and 5.

8. H.C.F. = 12, L.C.M. = 72, L.C.M. × H.C.F. = Product of two numbers.

9. (i) Yes, when two numbers are equal, say, 18, 18. (ii) No. For example, 15 and 18 have L.C.M. 90 and H.C.F. 3 so that H.C.F. < L.C.M.
(iii) Yes. Same example as in (i). 11. $a = b$.

12. $u = 2$, $v = -1$. The statement "a, b are co-prime" is superfluous.

13. (i) for all real values of x (ii) for $z = 0$ (iii) for $x = -1$.

14. $(0.25),^{-\frac{1}{3}} (0.065),^{-0\cdot5} \frac{1}{5}, -(1.1)^2$.

Form : Very Short Answer Type

1. $5, \frac{3}{2}, 0.023, -1$ are rational and $\sqrt{7}, \sqrt{12}, 2 + \sqrt{3}$ are irrational.

2. $0 < x < 1$ or $x < -1$. 3. $x > 1$ or $-1 < x < 0$.

4. $x = \pm 1$. 5. No. $3 + 2 = 5$ which is odd.

6. No 7. A rational number. 8. No.

9. Either $a=0$ or $b=0$ or both a and b are zeros. 10. 0.

11. 9, No.

Form : Objective Type

(i) *Multiple Choice type* :

1. (A). 2. (D). 3. (B).

4. (A). 5. (D). 6. (A).

(ii) *True-false type* :

1. T. 2. T. 3. F.

4. F. 5. F. 6. F.

7. T. 8. F. 9. F.

10. T. 11. T. 12. T.

13. T. 14. T. 15. F.

16. T. 17. F. 18. T.

19. F. 20. F. 21. T.

Matching type :

1. (1) – (D), (2) – (C), (3) – (A), (4) – (B).

2. (1) – (C), (2) – (E), (3) – (D), (4) – (A), (5) – (B).

UNIT II

FORM : **Essay Type**

2. (i) True (ii) True (iii) True (iv) False (v) False (vi) False (vii) False (viii) False (ix) True (x) True (xi) False (xii) False (xiii) True.

3. 6.

FORM : **Short Answer Type**

1. Elements of A is 0, there is no element of B, element of C is 5, elements of D are 5, − 5. There are no equal sets.

2. $\{ a \}, \{ b \}, \{ c \}, \{ a,b \}, \{ a,c \}, \{ b,c \}, \{ a,b,c \}, \varnothing$.

3. (i) $\{ 0,1,2,3,4,6,7,8,9 \}, \phi$. (ii) $\{ 0,1,2,3 \}, \{ 1,2 \}$
(iii) $\{ 3,5,6,9,10,12,15,20,25 \}, \{ 15 \}$.

4. $\{ (-2, 0), (-2,-5), (3,0), (3,-5), (8,0), (8,-5) \}$. 5. 9.

6. (i) $\{ 30,40,50,60,70,80,90 \}$ (ii) $\{ 30,40,50,60,70,80,90 \}$

7. (i) $\{ (a,1), (a,2), (b,1), (b,2), (c,1), (c,2) \}$
(ii) $\{ (1,a), (1,b), (1,c), (2,a), (2,b), (2,c) \}$
(iii) $\{ (a,1), (b,1), (c,1) \}$
No.

8. $x + \frac{3}{x}$. 9. $x - 4, \frac{1}{x} - 2$.

11. $xf(x)\, g(x) = -1$, No.

FORM : **Very Short Answer Type**

1. (i) 11 (ii) 5.   2. Yes.

3. (i) It is a set of all elements of A, which are not in B.
(ii) It is a set of all common elements of A,B,C.

4. (i) Not a set (ii) A set (iii) A set (iv) Not a set (v) Not a set (vi) A set.

5. (i) { 51,53,55,57,59,61,63,65,67,69,71,73,75,77,79 } (ii) { a,e }.

6. (i) Finite (ii) Finite (iii) Finite (iv) Infinite (v) Infinite.

7. { 3 }, { 5 }, { 3,5 }.   8. $a = b = c$ 9. 2.

10. . − 1.

FORM : **Objective Type**

(i) *Multiple choice type* :

1. (A).
2. (A).
3. (D).
4. (C).
5. (C).
6. (B).
7. (A).
8. (A).
9. (B).
10. (D).
11. (C).
12. (A).
13. (D).
14. (C).
15. (C).

(ii) *True false type* :

1. T.
2. F.
3. T.
4. F.
5. F.
6. F.
7. T.
8. T.
9. F.

10. T. 11. F. 12. F.

13. F. 14. T. 15. T.

16. F. 17. T. 18. T.

(iii) *Matching type* :

1. (1) – (D), (2) – (H), (3) – (F), (4) – (B), (5) – (G), (6) – (E), (7) – (C), (8) – (A).

(iv) *Sentence completion type* :

1. (A) $\cup$ (B) $\cap$ (C) $\cap$.

2. (A) $\subset$ (B) $\in$ (C) $\supset$.

4. $A \times (B \cap C)$. 5. does not form. 6. $(A \cap B) \cap C'$.

UNIT III

FORM : Essay Type

1. $x=2, y=1, -18$. 4. Geeta 24 years, Virendra 8 years.

5. The value of y in the first column is wrong. 6. $\frac{3}{4}$.

7. $x + 2y = 6$. 8. $x = a + b + c$.

10. P (3,7), Q (3,4), R $(\frac{3}{2}, 4)$. 11. $\frac{\sqrt{5}}{\sqrt{2}}$ v hrs.

FORM : Short Answer Type

1. $x = 0$. 2. (i) $x > 2$ (ii) $x \geqslant 1$. 3. 3. 4. 28.

6. (i) No solution (ii) No solution (iii) Has solution (iv) Has solution.

7. (a) $\{2\}$ (b) $\{4\}$ (c) $\{7,8,9\}$ (d) $\{1,2,3,4\}$ (e) ϕ (f) ϕ
(g) $\{2,3,4,5,6,7,8,9\}$.

8. (i) $x = -\frac{4}{3}$ (ii) $x = \frac{3}{4}$

9. (i) $\left.\begin{array}{l} x = 4 \\ y = 2 \end{array}\right\} \left.\begin{array}{l} x = 2 \\ y = 3 \end{array}\right\} \left.\begin{array}{l} x = 0 \\ y = 4 \end{array}\right\}$

(ii) $\left.\begin{array}{l} x = 0 \\ y = 0 \end{array}\right\} \left.\begin{array}{l} x = 0 \\ y = 1 \end{array}\right\} \left.\begin{array}{l} x = 1 \\ y = 0 \end{array}\right\} \left.\begin{array}{l} x = 1 \\ y = 1 \end{array}\right\} \left.\begin{array}{l} x = 2 \\ y = 0 \end{array}\right\} \left.\begin{array}{l} x = 3 \\ y = 0 \end{array}\right\}$

10. $x > -\frac{3}{2}$. 11. 2.

12. (i) $x < 2$ and $y < 1$ such that $5x - 2y = 8$ (ii) $x < 2$ and $y < -1$ such that $x - y = 3$.

FORM : **Very Short Answer Type**

1. $(-1, 0)$.
2. Third quadrant.
3. Right side.
4. No.
5. 9m.
6. 2.
7. (i) Dependent (ii) Inconsistent.
8. $x + y \leqslant 11$.
9. $2x < y$ and $x + y = 9$, where x is the number of Bengali books and y is the number of English books.
10. 4.
11. Yes, if the pair of resulting equations is consistent, otherwise not.
12. 2,2.
13. No.
14. No.

FORM : **Objective Type**

(i) *Multiple choice type* :

1. (C).
2. (D).
3. (C).
4. (C).
5. (B).
6. (C).
7. (D).
8. (B).
9. (D).
10. (C).
11. (D).
12. (C).
13. (C).

(ii) *True-false type* :

1. T.
2. F.
3. F.
4. T.
5. F.
6. F.
7. T.
8. F.
9. F.
10. T.
11. F.
12. F.
13. F.
14. T.

(iii) *Matching type* :

1. (1) — (D), (2) — (C), (3) — (A), (4) — (E), (5) — (B).
2. (1) — (B), (2) — (F), (3) — (E), (4) — (A), (5) — (C), (6) — (D).

(iv) *Sentence completion type* .

1. (A) $<$ (B) $=$ (C) inconsistent (D) second.
2. (A) $<$ (B) $>$ (C) $>$. 3. (A) one (B) two. 4. (A) 4π (B) 4.

UNIT IV

FORM : **Essay Type**

1. 3962.00.
2. 301.8 sq. cm.
3. 5,3.
4. 47.
5. $\left.\begin{array}{l} x = 20 \\ y = 5 \end{array}\right\}$ or $\left.\begin{array}{l} x = 5 \\ y = 20 \end{array}\right\}$

11. —.9999, .1000, $\frac{0\ 598 \times 9.87}{59}$ or similar expressions.
12. 0.44.

FORM : **Short Answer Type**

1. $\bar{4}.1761$.
2. 4 log 2 + log 3.
3. 1.710.
5. $\log \frac{x}{y}$.
6. 0.4771.
7. 6.
8. 0.
9. 5.
10. ± 1.
11. 100.
12. 20, — 5.
13. 6 and — 12.
14. m and hence m^n must be positive. Example : $\log 16 = \log (-4)^2 = 2\log(-4)$ which is not real. Hence, log 16 is not real, which is absurd.

17. No.

FORM : **Very Short Answer Type**

1. 0.
2. .4771.
3. n—1.
4. $p^2 + q^2$.
5. 1.4771.
6. $2 \log \sqrt{15}$
8. 1.
9. 2 cm., yes., log 2 = .3010 is superfluous,

10. 10 sq. cm.

FORM : **Objective Type**

(i) *Multiple choice type* :

1. (D). 2. (B). 3. (B).

4. (B). 5. (A). 6. (C).

7. (B). 8. (A). 9. (B).

(ii) *True false type* :

1. F. 2. F. 3. F.

4. T. 5. F. 6. T.

7. T. 8. F. 9. T.

10. F. 11. F. 12. T.

13. T.

(iii) *Matching type* :

1. (1) — (B), (2) — (D), (3) — (E), (4) — (C), (5) — (F), (6) — (A).

2. (1) — (B), (2) — (A), (3) — (D), (4) — (C).

3. (1) — (F), (2) — (D), (3) — (C), (4) — (A), (5) — (B), (6) — (G), (7) — (E).

4. (1) — (D), (2) — (E), (3) — (B), (4) — (C), (5) — (A).

5. (1) — (C), (2) — (D), (3) — (A), (4) — (B).

(iv) *Sentence completion type* :

1. (A) 0 (B) np (C) n^p (D) $a = \frac{1}{b}$.

2. (A) $b^x = a$ (B) $m > 0$. 3. 4. 4. .9030.

(v) *Only wrong answer type* :

1. (C). 2. (D). 3. (D).

4. (A). 5. (B).

UNIT V

FORM : Essay Type

1. Rs. 2.00.
3. Rs. 11500.00.
4. Rs. 4800.00.
5. 21%.
6. Rs. $33\frac{1}{3}\%$.
7. Rs. 1000.00.
8. Rs. 820.00
9. Rs. 140.00.
10. No. Loss = Rs. 50.00

FORM : Short Answer Type :

11. Rs. 71.25.
12. Rs. 80.00.
13. Rs. 800.00.
14. 2 years.
15. Rs. 3060.00.
16. 44% gain.
17. 3 for Rs. 11.00.
18. Re. 0.50.
19. Rs. 3.41.
20. Case I : Re 0.75, Case II : Re 1.87, Case II.
21. Rs. 11000.00.
22. $\frac{100}{9}\%$
23. Rs. 144. 00.
24. Rs. 1284.00.
25. 12100.
26. Rs. 3.20 per kg.
27. 4 years.

FORM : Very Short Answer Type

1. $\frac{25}{4}\%$.
2. Rs. 2520.00.
3. Rs. 26.25.
4. 16 years.
5. 2r%.
6. Amount $= P\left(1+\frac{r}{400}\right)^{4n}$
7. 60 %.

FORM : **Objective Type**

(i) *Multiple choice type* :

1. (A).
2. (B).
3. (C).
4. (B).
5. (B).
6. (A).
7. (C).

(ii) *True false type* :

1. T.
2. F.
3. T.
4. F.
5. F.
6. T.
7. F.
8. T.
9. F.
10. T.
11. T.
12. T.
13. F.

(iii) *Sentence completion type* :

1. Re. 0.25.
2. 10%.
3. 28%.
4. $0.1\,xy$.
5. $y - xy$.
6. 0.05.
7. 1.05^2.
8. 2P.
9. a.
10. 1.
11. P.
12. Rs. 44.00.
13. 20%.

UNIT VI

FORM : Essay Type

1. $\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 2.$

4. 52°30′, 7°30′.

7. $\frac{\pi}{4}.$

9. $\pm$ m.

10. $2(\sqrt{3}+1)^{m}.$

11. $\frac{10}{3}(3-\sqrt{3})$ metres.

12. $\frac{\sqrt{3}-1}{2\sqrt{2}}.$

FORM : Short Answer Type

3. $2\sqrt{3}.$

5. $\frac{2mn}{m^2+n^2}.$

7. $(\sin\alpha-\cos\alpha)^2.$

10. 8 metres.

12. $\pm 1.$

13. $B = 90° - A.$

17. $\frac{7}{25}.$

18. $\frac{al+bm}{al-bm}.$

FORM : Very Short Answer Type

1. (a) (i) (b) (iii) (c) (iv) (d) (ii).

2. $\frac{\sqrt{5}-1}{4}.$

3. $\frac{\sqrt{5}+1}{4}.$

4. 90°.

6. 45°

7. 1.

8. No.

9. No, since $\sin\theta+\cos\theta \leqslant \sqrt{2}.$

10. No, since $\sin^2\theta+\cos^2\theta = 1.$

11. $2\sqrt{3}$ cm.

12. $\frac{1}{\sqrt{3}}.$

Form : **Objective Type**

(i) *Multiple choice type* :

1. (B).
2. (A).
3. (A).
4. (D).
5. (C).
6. (D).
7. (C).

(ii) *True false type* :

1. F.
2. T.
3. T.
4. F.
5. F.
6. F.
7. T.
8. F.
9. T.
10. T.
11. F.
12. T.
13. T.
14. F.

(iii) *Matching type* :

1. (1) — (B), (2) — (C), (3) — (D), (4) — (A).
2. (1) — (B), (2) — (A), (3) — (D), (4) — (C).
3. (1) — (D), (2) — (C), (3) — (B), (4) — (A).

(iv) *Sentence completion type* :

1. x.
2. $45°$.
3. 1.
4. a right-angled.
5. $45°$.
6. less than or equal to $\sqrt{2}$.
7. $\frac{4ab}{(a+b)^2}$ $(a \neq b)$.
8. $\sin x$ and $\frac{1}{\sqrt{2}} \cos y$.

UNIT VII

FORM : **Short Answer Type**

1. 95° 2. 100°, 80°, 100°.

4. Alternate angles are : $\angle 4$ and $\angle 6$, $\angle 3$ and $\angle 5$, $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$. Corresponding angles are : $\angle 2$ and $\angle 6$, $\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 8$, $\angle 3$ and $\angle 7$.

5. $\angle XYP = 130^\circ$ $\angle YZS = 50^\circ$, $\angle SZB = \angle 130^\circ$.

6. 120°, 60°, 120°, 7. 120°. 8. 90°, 90°, 90°, 90°.

10. 111°, 51°, 18°, an obtuse-angled triangle or a scalene triangle.

12. 15. 13. $AB \parallel CD$. 14. $AB \parallel CD$.

17. A line through B parallel to AC.

19. The three parts should be placed along a straight line such that $\angle A + \angle B + \angle C = 180^\circ$ (a straight angle).

20. 30°.

FORM : **Very Short Answer Type**

1. $AB = AC$. 2. 86°. 3. One.

4. 120°. 5. 540°, no. 6. 180

7. One. 8. (i) No. (ii) No (iii) Yes (iv) No.

9. 115°. 10. As many as one likes. 11. 360°, no,

12. $\angle B - \angle C$ or $\angle C - \angle B$. 13. 18°.

14. $(2x - 4)\,90 = xy$.
15. They will be parallel.

17. $\angle CAD$ and $\angle DCB$, $\angle ACD$ and $\angle DBC$.

Form : Objective Type

(i) *Multiple choice type* :

1. (C).
2. (B).
3. (A).
4. (D).
5. (A).
6. (C).

(ii) *True-false type* :

1. F.
2. T.
3. F.
4. F.
5. T.
6. T.
7. F.
8. T.
9. F.

(iii) *Sentence completion type* :

1. more than three.
2. more than four.
3. supplementary.
4. an external bisector.
5. a single.
6. parallel.
7. right isosceles.
8. =.
9. =.
10. a right angle.
11. a right angle.
12. 60°.

UNIT VIII

Form : Essay Type

16. 7.75 cm.

Form : Short answer type

1. AC = DF or ∠ABC = ∠DEF; SSS, ASA, SAS.

8. No, since BC and DY are not corresponding sides.

9. (i), (iii), (v), (vi).   14. $\frac{1}{2}$ (OA + OB)

Form : Very Short Answer Type

1. ∠B = ∠E.   2. No.

3. AB = DE or BC = EF or AC = DF.   4. 90°.

5. Rhombus.   6. AC = 6cms., BD = 4cms.

7. 2 : 1.   8. No.   9. (i) Yes (ii) No.

10. 1 : 1.   11. Rhombus.   12. AB ∥ CD.

13. x = 2.   14. 1 : 1 : 2.   15. No.

16. No. In that case, the sum of the interior angles will exceed 4 right angles, which is absurd.

17. Yes.

18. (i) The two diagonals are perpendicular to each other in both the cases.
    (ii) Each diagonal bisects the angle through which it passes in both the cases.

19. 150°, 30°, 150°, 30°. 20. No. 21. Yes, 90°.

22. Yes; 45°, 135°, 45°, 135°.

FORM : Objective Type

(i) *Multiple choice type* **:**

(a) *Only correct answer type* **:**

1. (A). 2. (C). 3. (C).

4. (B). 5. (D). 6. (C).

7. (C). 8. (C).

(b) *Only wrong answer type* **:**

1. (C). 2. (D).

(c) *Mixed answer type* **:**

1. (A) A (B) S (C) A (D) A (E) S (F) N.

2. (A) S (B) S (C) S (D) S (E) N (F) A.

3. (A) A (B) N (C) S (D) N (E) S.

(i) *True-false type* :

1. F. 2. T. 3. F.

4. F. 5. F. 6. F.

7. T. 8. F. 9. T.

10. T. 11. F. 12. F.

13. T. 14. T. 15. F.

16. T. 17. T.

(iii) *Matching type* :

1. (1) — (E), (2) — (F), (3) — (D), (4) — (B), (5) — (A), (6) — (C).
2. (1) — (B), (2) — (D), (3) — (A), (4) — (C).
3. (1) -- (C), (2) — (B), (3) — (D), (4) — (A).
4. (1) — (B), (2) — (D), (3) — (A), (4) — (C).
5. (1) — (D), (2) — (C) (3) — (B), (4) — (A).

(iv) *Sentence completion type* :

1. more than one.
2. necessarily.
3. greater than.
4. a right angle.
5. rhombus.
6. congruent.
7. supplementary.
8. rectangle.
9. BAQ.
10. (b) and (c).
11. rectangle.
12. two.
13. a unique.

UNIT IX

FORM : Essay Type

4. 48°, 96°, 42°, 42°, 132°.

5. No.

9. (i) $\angle PXA = \angle AYQ$ (ii) $\angle PXA + \angle AYQ = 180°$.

FORM : Short answer type :

1. As many as one likes.

2. As many as one likes.

3. Yes.

4. No.

6. Angles are equal.

7. Values of the angles are superfluous.

8. No.

9. No.

11. Yes.

12. No.

14. Rectangle.

16. $u=44$, $v=43$, $x=19$, $y=74$, $z=43$, $t=19$.

19. 35°.

20. 360°.

23. Yes.

25. $\frac{1}{2}$ min., $2(\angle CAB + \angle DBA) - 180°$.

FORM : Very Short Answer Type

1. No.

2. 1 : 1 : 1.

3. 30°.

4. 90°.

5. 60°.

6. 60°.

7. No.

8. $x = 2y$.

9. Yes.

10. No.

11. 10 cm.

12. Yes.

13. Outside.

14. $\frac{a}{\sqrt{2}}$ cm.

Form : **Objective Type**

(i) *Multiple choice type* :

1. (B).
2. (B).
3. (A).
4. (B).
5. (A).
6. (B).
7. (D).
8. (D).
9. (B).
10. (B).

(*ii*) *True-false type* :

1. F.
2. F.
3. T.
4. T.
5. F.
6. T.
7. F.
8. T.
9. T.
10. F.
11. T.
12. F.
13. T.
14. T.
15. T.
16. T.

(iii) *Sentence completion type* :

1. always equal.
2. a right angle.
3. supplementary.
4. less than 100°.
5. less than 70°.
6. a right-angled triangle.
7. a diameter.
8. less than r^2.

UNIT X

FORM : **Essay type**

20. 2. 25. 2.

FORM : **Short answer Type**

1. (i) A circle with the given point as the centre and r as radius. (ii) The perpendicular bisector of the line segment joining the given points.

3. 180°. 4. PI is the internal bisector of ∠ QPR.

8. They are equal. 9. No. 10. AB > BD and AC > CD.

12. The statement is true. 15. AB > AC.

FORM : **Very short Answer Type**

1. Centroid. 2. x cm. 3. Orthocentre.

4. (i) Yes (ii) No (iii) No 5. No

6. (i) No (ii) Yes. 7. No; 60°.

FORM : **Objective Type :**

(i) *Multiple choice type* :

1. (B). 2. (C). 3. (B).

4. (D). 5. (C). 6. (B).

7. (A).

(ii) *True false-type* :

1. F.
2. F.
3. T.
4. T.
5. F.
6. T.
7. T.
8. T.
9. F.
10. T.
11. T.
12. F.
13. F.
14. F.
15. T.
16. T.
17. T.
18. F.
19. T.
20. T.
21. F.
22. F.

(iii) *Sentence completion type* :

1. centroid.
2. 4 cm.
4. an acute angle.
5. an angular bisector of $\angle A$.
6. $180^\circ - \frac{A + B}{2}$.
7. 70°.
8. not a centroid of the triangle.
9. a square
10. isosceles right-angled.
11. circumcentre.
12. touch at one point.
13. 1 : 1.
14. not be equal.
15. not be equal.

BIBLIOGRAPHY

1. AHMANN, J.S., GLOCK, M.D. AND WARDEBERG, H.L. — Evaluating Elementary School Pupils, Boston, 1960.

2. BAUERNFEIND, R.H. — Building a School Testing Programme, Houghton Mifflim, Boston, 1963.

3. BEAN, K.L. — Construction of Educational and Personnel Tests, Mc-Graw Hill Book Company, Inc., New York, 1953.

4. BLOOM, B.S. (*Editor*) — Taxonomy of Educational Objectives, Handbook I Cognitive Domain, Longmans Green & Co., New York, 1956.

5. BLOOM, B.S. *et al* — Handbook of Formative and Summative Evaluation of Pupils Learning, Mc-Graw Hill, 1971.

6. DANIELS, J.C. — Teacher's Handbook of Test Construction, Marking and Records, Crosby Wood & Sons Ltd., London, 1949.

7. DUTTON, W.H. — Evaluating Pupil's Understanding of Arithmetic, Prentice Hall, New Jersey, 1964.

8. GERBERICH, J.R. — Specimen Objective Test Items (A Guide to Achievement Test Construction), Longmans Green & Co., New York, 1954.

9. HENRY, N.B. — The Measurement of Understanding, 45th Year Book of NSSE, Chicago, 1964.

10. INGENKAMP KARTHEINZ (*Editor*) — Developments in Educational Testings, Vols I and II, University of London Press Ltd., 1969.

11. MANZEL, E.W. — Suggestions for the use of New Type Tests in India, Oxford University Press, Bombay, 1945.

12. ODELL, C.W. — How to Improve Class-room Testing, Dubsides, Lows; William C. Brown, 1953.

13. TOMAS, R.N. — Judging Student Progress, Longmans Green & Co., 1960.

14. TRAVERS, R.W.M. — How to Make Achievement Tests, The Odyssey Press, New York, 1950.

15. TRASELER, A.E., JACOBS, ROBERT, SELOVER *et al* — Introduction to Testing and the use of Test Results in Public Schools, Harper and Brothers, New York, 1958.

16. VERNON — Measurement of Human Abilities, Methuen.

17. WOOD, D.A. — Test Construction, Columbus, Ohio, Charles E. Merrill Books, 1960.

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